Mining Massive Datasets

Lecture 7

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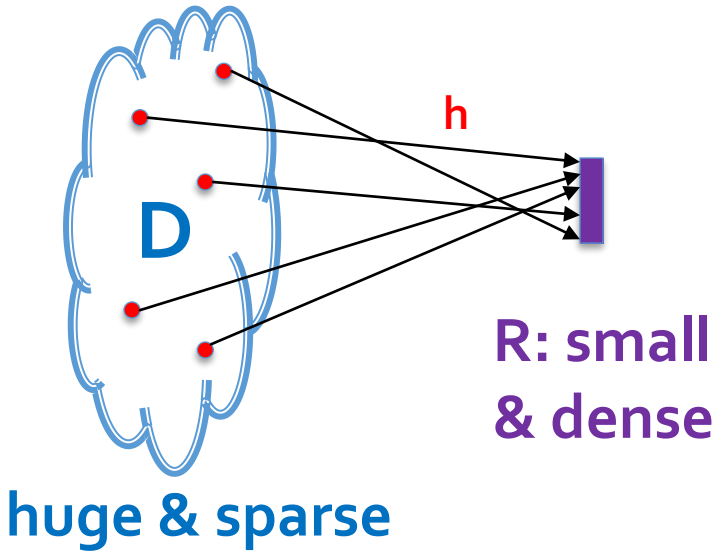
Note on Slides

A substantial part of these slides come (either verbatim or in a modified form) from the book Mining of Massive Datasets
by Jure Leskovec, Anand Rajaraman, Jeff Ullman (Stanford University).
For more information, see the website accompanying the book: http://www.mmds.org.

A Word on Hash Functions and Data Structures

Hash Functions /1

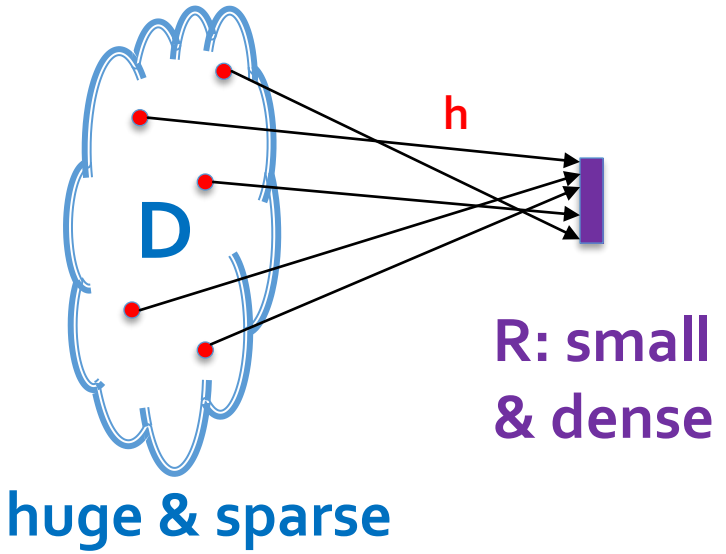
- A **hash function h: D→R** maps objects from a (usually huge) domain space **D** to a (smaller) range **R** of consecutive integers ("**buckets**")
  - E.g. **D** = set of all files in HDFS, **R = [0,…,b-1]**, b=10000
- Intuitively, h computes a short **signature** of an object
- **h** is in general not injective => **collisions** possible (i.e. h(x)=h(y) for x ≠y)



Hash Functions /2

- Another view: mapping from a sparse storage for D to dense storage (for R)
- A table containing all possible objects in D would be too large, but a table for all buckets in R is possible
- Implementation?

- **x** $\in$D is treated as integer
- **h(x) = x mod b** (b preferably a prime)

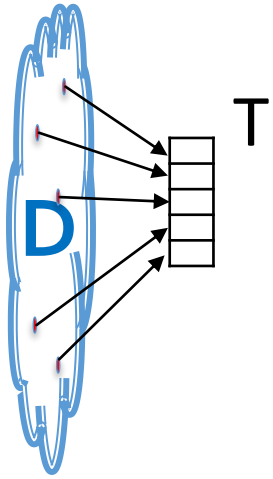


HashSet /HashMap, Dictionary

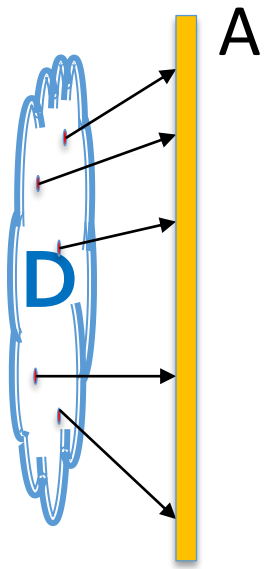
- Problem of storing and fast search for elements is very common => optimized data structures
- Java
  - HashSet: a set implementation
  - HashMap: for storing pairs <key, value> and fast finding of values by a key
  - TreeMap: also for <key, value>, use balanced trees
- Python, C++ (STL):
  - dictionary, uses hashing

Example: Implementing Sets /1

Given a set **M** $\subseteq$ D, we want to test: *is* $q \in M$?



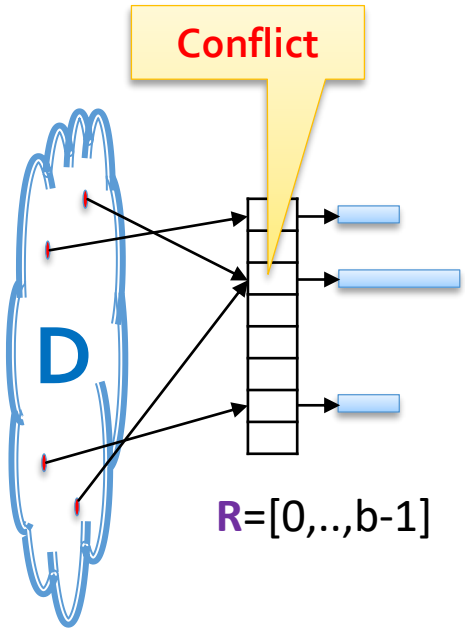
- Solution 1: sorted table
  - Storage: sort all existing elements by their IDs, store IDs (or references) in a table T
  - Query for element q: binary search for q.ID in T
    - Assume that q is like a number, e.g. a bit sequence



- Solution 2: bit array
  - Storage: Create a huge bit array A covering whole D and set A[x.ID] = 1 iff x $\in$ M
  - Query for q: make a lookup A[q.ID]
    - *True* iff q is in the set M
  - Very fast but we need a huge and sparse array (e.g. elements = strings of fixed length)

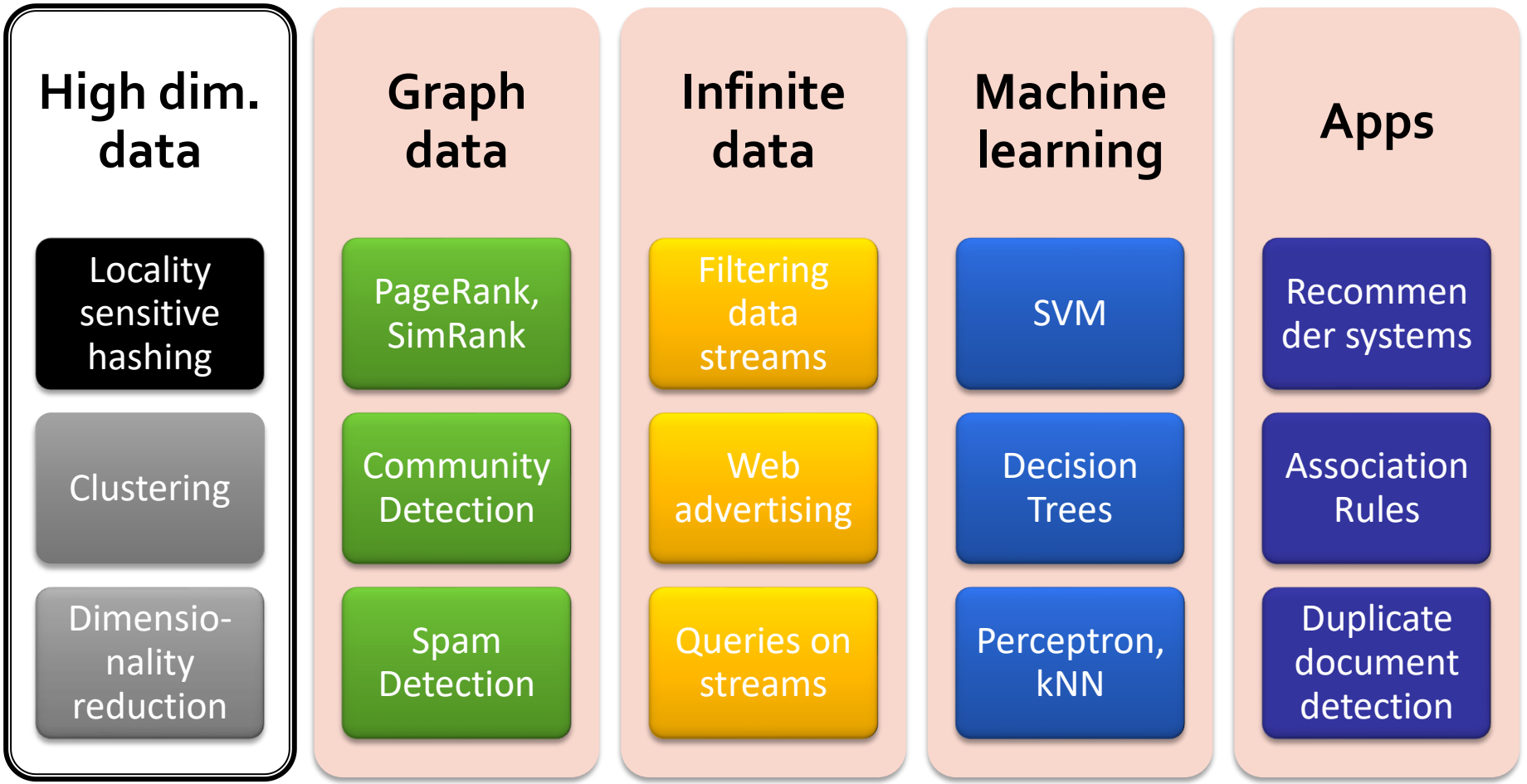
Example: Implementing Sets /2

- Solution 3: hash table
- Storage and setup:
  - Fix an integer b ≈ c* |**M**| and a hash fn **h**:D→**R**, **R** = [0,..,b-1]
  - Create an array **T** of size b with lists (at first: empty lists)
  - If x ∈ **M**: update list L = **T**[**h**(x)] by adding x (or a ref) to L
- Query for element q:
  - Make a lookup L= **T**[**h**(q)]; if list L not empty, check q ∈ L => yes iff q ∈ **M**
- Runtime, space requirements?
  - Runtime: fast, O(1) for search and inserting
  - Space: b references to lists, b lists, plus approx. size of all elements in M



Finding Similar Items: Locality Sensitive Hashing

Locality Sensitive Hashing



Programming in Spark & MapReduce

A Common Metaphor

- Many problems can be expressed as finding "similar" objects:
  - Find near-neighbors in high-dimensional space
- Examples:
  - Points in the same cluster (clustering)
  - Pages with similar words
    - For duplicate detection, classification by topic
  - Customers who purchased similar products
    - Products with similar customer sets

Problem for Today's Lecture

- **Given: High dimensional data points $x_1, x_2, \ldots$**
  - **For example:** Image is a long vector of pixel colors

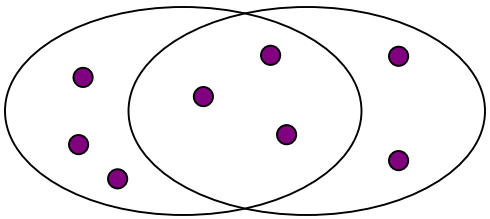
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow [1\ 2\ 1\ 0\ 2\ 1\ 0\ 1\ 0]$$

- And some distance function $d(x_1, x_2)$
  - Which quantifies the "distance" between x_1 and x_2
- **Goal:** Find **all pairs of data points** (x_i, x_j) that are within some distance threshold $d(x_i, x_j) \leq s$
- **Note:** Naïve solution would take $O(N^2)$ ☹

  (N = #data points), see hierarchical clustering
- **MAGIC**: This can be done in $O(N)$!! How?

Distance Measures

- **Goal:** Find near-neighbors in high-dim. space
  - We formally define "near neighbors" as points that are a "small distance" apart
- We need a "distance" metric
- E.g. Jaccard distance/similarity (recall)
  - The **Jaccard similarity** of two sets is the size of their intersection divided by the size of their union: $sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$
  - **Jaccard distance**: $d(C_1, C_2) = 1 - |C_1 \cap C_2| / |C_1 \cup C_2|$



3 in intersection
8 in union
Jaccard similarity= 3/8
Jaccard distance = 5/8

Task: Finding Similar Documents

- Goal: Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs
- Applications:
  - Mirror websites, or approximate mirrors
    - Don't want to show both in search results
  - Similar news articles at many news sites
    - Cluster articles by "same story"
- Problems:
  - Too many documents to compare all pairs
  - Documents are so large or so many that they cannot fit in main memory
  - Many small pieces of one document can appear out of order in another

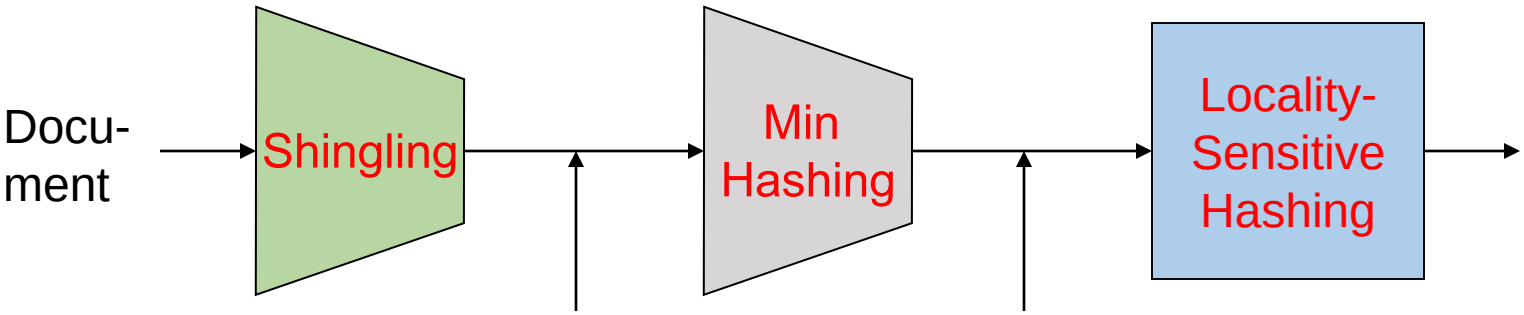
Motivation for a Fast Algorithm

- Suppose we need to find near-duplicate documents among $\mathrm{N} = 1$ million documents
- Naïvely, we would have to compute pairwise Jaccard similarities for every pair of docs
- How long (at 10^6 comparisons/sec)?
  - $N(N-1)/2 \approx 5*10^{11}$ comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take 5 days
- For $N = 10$ million, it takes more than a year…

3 Essential Steps for Similar Docs

1. ***Shingling:*** Convert documents to sets
2. ***Min-Hashing:*** Convert large sets to short signatures, while preserving similarity
3. ***Locality-Sensitive Hashing:*** Identify pairs of signatures likely to be from similar documents
4. ***Final filtering:*** We get (few) candidate pairs! => Those can be checked by pairwise comparisons

The Big Picture



The set of strings of length ***k*** that appear in the document

Signatures: short integer vectors that represent the sets, and reflect their similarity

Candidate pairs: those pairs of signatures that we need to test for similarity

Shingling

Documents as High-Dim Data

- Step 1: *Shingling:* Convert documents to sets
- Simple approaches:
  - Document = set of words appearing in document
  - Document = set of "important" words
- Don't work well for this application; Why?
- Need to account for ordering of words!
- A different way: Shingles!

Define: Shingles

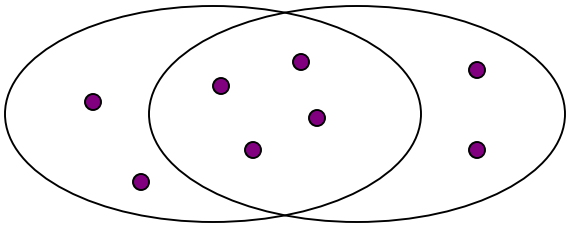
- A ***k*-shingle** (or ***k*-gram**) for a document is a sequence of *k* (consecutive) tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
- Assume e.g.: tokens = characters

- **Example:** k=2; document D_1 = abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}
  - **Option:** Shingles as a bag (multiset), count "ab" twice: $\mathbf{S'(D_1)}$ = {ab, bc, ca, ab}

Similarity Metric for Shingles

- Now: doc. D_1 is a set of its k-shingles: $C_1=S(D_1)$
- Equivalently, each document is a 0/1 vector in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse
- For sets, a natural similarity measure is the **Jaccard similarity:**

$$sim(D_1, D_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$$



Working Assumption

- Documents that have lots of shingles in common have similar text (content)
  - Even if the text appears in different order
- **Caveat**: You must pick *k* large enough, or most documents will have most shingles
  - *k* = 5 is OK for short documents
  - *k* = 10 is better for long documents

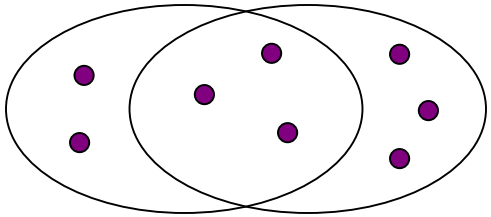
Compressing Shingles (Optional)

- To compress long shingles, we can **hash** them to (say) 4 bytes (value of a hash function)
- => We can represent a document by the set of hash values of its *k*-shingles
- Example: k=2; document D_1= abcab:
  - Set of 2-shingles: $S(D_1)$ = {ab, bc, ca}
  - Set of hash val's of the shingles e.g.: {1, 5, 7}
  - **Note:** Non-bijective, i.e. two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared

MinHashing

Minhashing: Converting **large sets** to **short signatures**, while **preserving similarity**

Encoding Sets as Bit Vectors



- Many similarity problems can be formalized as **finding subsets that have significant intersection**
  - Encode sets using 0/1 (bit, Boolean) vectors
  - Interpret set intersection as bitwise **AND**, and set union as bitwise **OR** => fast
- **Example:** $\mathbf{C_1}$ = 10111; $\mathbf{C_2}$ = 10011
  - Size of intersection **= 3**; size of union **= 4**
  - **Jaccard similarity** (not distance) **= 3/4**
  - **Distance: $d(C_1,C_2)$ = 1 – (Jaccard similarity) = 1/4**

From Sets to Boolean Matrices

- **Rows** = elements (shingles)
- **Columns** = sets (documents)
  - 1 in row ***e*** and column ***s*** if and only if ***e*** is a member of ***s***
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1)*
  - **Typical matrix is sparse!**
- **Each document is a column:**
  - **Example: sim(C_1 ,C_2) = ?**
    - Size of intersection = 3; size of union = 6, Jaccard similarity (not distance) = 3/6
    - **d(C_1,C_2) = 1 – (Jaccard similarity) = 3/6**

Documents

Shingles

1	1	1	0
1	1	0	1
0	1	0	1
0	0	0	1
1	0	0	1
1	1	1	0
1	0	1	0

Outline: Finding Similar Columns

- **So far:**
  - Documents $\rightarrow$ Sets of shingles
  - Represent sets as Boolean vectors in a matrix
- **Next goal:** Find similar columns from small (similar) signatures
  - Similarity of columns == similarity of signatures

What to do with small signatures?

- **(Recall) Next Goal:** Find similar columns from small signatures
- **Naïve approach:**
  - **1) Signatures of columns:** small summaries of columns
  - **2) Examine pairs of signatures** to find similar columns
    - **Essential:** Similarities of signatures and columns are related
  - **3) Optional:** Check that columns with similar signatures are really similar
- **But ..** comparing all pairs of signatures may take too much time: **Job for LSH**
  - Later

Hashing Columns (Signatures)

- **Key idea:** map each column ***C*** to a small ***signature h(C)***, such that:
  - **(1)** ***h(C)*** is small enough that the signature fits in RAM
  - **(2)** ***sim(C_1, C_2)*** is the same as the "similarity" of signatures ***h(C_1)*** and ***h(C_2)***

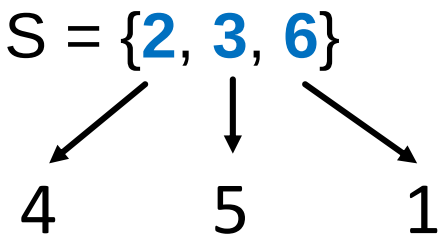
- **Goal: Find a hash function *h(·)* such that:**
  - If ***sim(C_1,C_2)*** is high, then with high prob. ***h(C_1) = h(C_2)***
  - If ***sim(C_1,C_2)*** is low, then with high prob. ***h(C_1) ≠ h(C_2)***

Min-Hashing

- **Goal: Find a hash function $h(\cdot)$ such that:**
  - if $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - if $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$
- Clearly, the hash function **depends** on the similarity metric $sim(C_1,C_2)$
  - Not all similarity metrics have a suitable hash function
- But there is a suitable hash function for the Jaccard similarity: it is called **MinHash**

MinHash: Definition

- Def.: Let **h** be a hash function that maps the members of S to distinct integers, and for any set S define $\mathbf{MinHash_h}(S) = \mathbf{h_{min}}(S)$ to be the minimum value of **h**(x)
  - **Example**:
    - Assume S = {**2**, **3**, **6**} and
    - **h**(**2**) = 4, **h**(**3**) = 5, **h**(**6**) = 1



  - What is $\mathbf{h_{min}}(S)$?
  - Of course, $\mathbf{h_{min}}(S) = 1$

Min-Hashing: Using Permutations

- Recall: we represent each set as a Boolean vector **C** (here: S = {**2**, **3**, **6**})
- Assume that a hash function **h** is given by a (random) permutation π of the rows of the Boolean vector
  - **h** fulfills: "… maps the members of S to distinct integers"

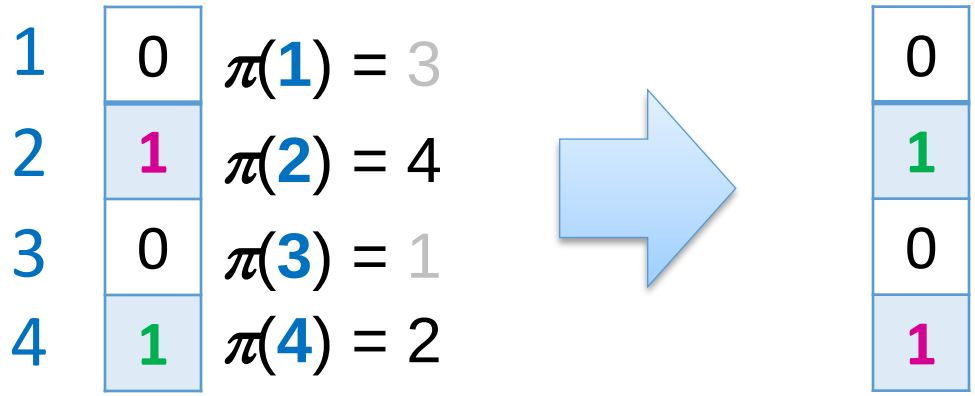
		Row perm.
1	0	$\pi(1) = 3$
2	1	$\pi(2) = 4$
3	1	$\pi(3) = 5$
4	0	$\pi(4) = 6$
5	0	$\pi(5) = 2$
6	1	$\pi(6) = 1$

- Then $\mathbf{MinHash_h}(S)$ is the index of the **first row** of the permuted column ***C*** (= rows ordered by vals of π) with value **1**
- => Again, $\mathbf{h_{\pi,min}}(S) = 1$

1	1
2	
3	
4	1
5	1
6	

Example of Using Permutations

- "Goes to" representation of a permutation:
  - Original row with index **r** goes to row π(**r**)
- Recall: Then **MinHash$_h$**(S) is the index of the first row of the permuted column ***C*** with value **1**



- ***Thus, $h_{min, \pi}(C)$*** = 2

Min-Hashing on Matrices

- Recall:
  - **Rows** = elements (shingles)
  - **Columns** = sets (documents)
    - 1 in row ***e*** and column ***s*** if and only if ***e*** is a member of ***s***
- For each column (= set):
  - We use several (e.g., 100) independent hash functions (that is, permutations) to create a **signature of a column**

Documents

Shingles

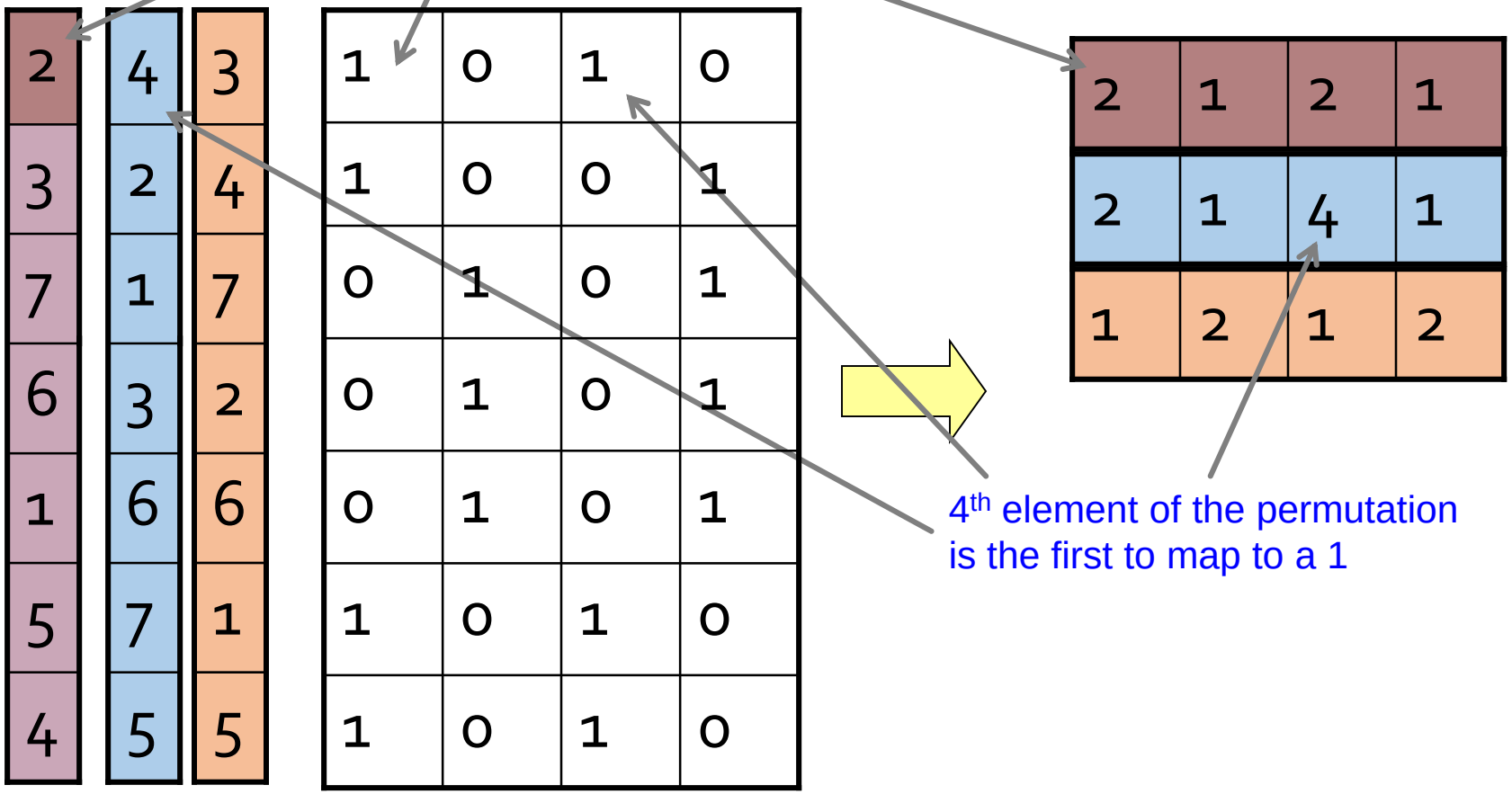
1	1	1	0
1	1	0	1
0	1	0	1
0	0	0	1
1	0	0	1
1	1	1	0
1	0	1	0

Min-Hashing Example

Value **n** at position **p** means: "previous row **p** goes to row **n** in the new ordering"

2nd element of the permutation is the first to map to a 1

Permutation π | **Input matrix (Shingles x Documents)** | **Signature matrix *M***



4th element of the permutation is the first to map to a 1

Min-Hashing Example

Value n at position p means: "previous row p goes to row n in the new ordering"

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1		
1	0		
1	0		

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Heuristic for computing minhash for a permutation specification „column" P and an input X:
Set k= 1

1. In P, search for row r with value k.
2. Is X[r] == 1? Yes => Minhash is r, finished.
3. No => k := k+1, repeat the loop from #1.

The Min-Hash Property

- **Choose a random permutation π**

- **Claim:** **$\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$**
  - I.e. the probability that the minhash-values $h_\pi(C_1)$ of C_1 and $h_\pi(C_2)$ of C_2 **agree** (under the permutation π)
  - is exactly
  - the **Jaccard-similarity** of C_1 and C_2 .

"Proof": Four Types of Rows

- Given cols C_1 and C_2, rows may be classified as:

	C_1	C_2
A	1	1
B	1	0
C	0	1
D	0	0

a = num rows of type A,
b = num rows of type B,
c = etc.
Disjunctive row sets!

- **Note:** Jaccard-sim. is: **$sim(C_1, C_2) = a/(a+b+c)$**
- **Then: Pr**$[h(C_1) = h(C_2)] = Sim(C_1, C_2)$
  - Go down the permuted cols C_1 and C_2 until we see a 1 (in any of both columns)
  - There are a+b+c rows at which we can stop
  - The prob. that we stopped at type-A row is a/(a+b+c)
  - **If it's a type-*A* row, then $h(C_1) = h(C_2)$;** $h(C_1) \neq h(C_2)$ for types B/C

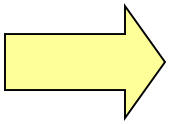
Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0



Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Similarity for Signatures

- We know: $\mathbf{Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)}$
- We are interested in $\mathbf{sim(C_1, C_2)}$
  - => We need to estimate $\mathbf{Pr[h_\pi(C_1) = h_\pi(C_2)]}$
- But testing $\mathbf{h_\pi(C_1) = h_\pi(C_2)}$ gives us only true or false!

- **Idea (similar to Monte-Carlo methods):**
  - **Given a random variable X with values 0, 1 and a distribution (1-p, p) : sample X many times to estimate p!**
  - => Use many different min-hash functions and compute the fraction of cases in which they agree

- Definition: the *similarity of two signatures* is the fraction of the hash functions in which they agree
- This approximates $\mathbf{sim(C_1, C_2)}$

Min-Hash Signatures

- **Pick K=100 random permutations of the rows**
- Think of ***sig*(C)** as a column vector
  - ***sig*(C)[i] =** the index of the first row that has a 1 in column *C,* according to the *i*-th permutation, i.e.

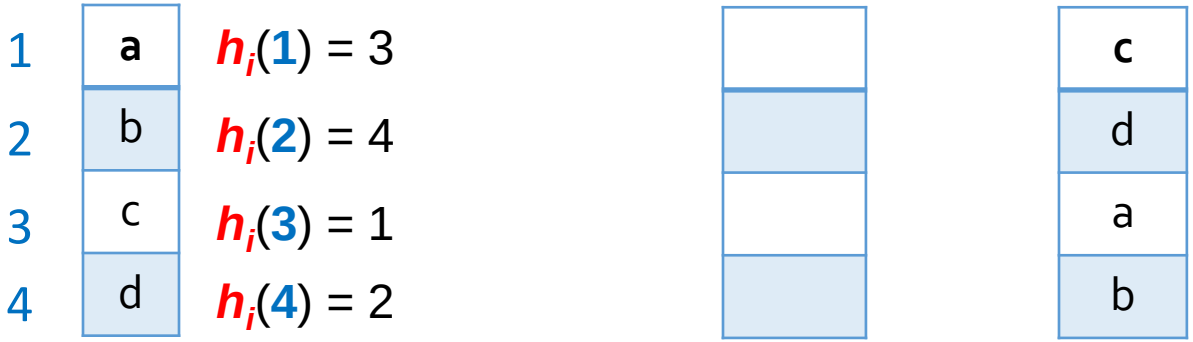
$$sig(C)[i] = \min(\pi_i(C))$$

- **Note:** The sketch (signature) of document *C* is small **$\sim$100 numbers!**

- We achieved our goal! We "compressed" long bit vectors into short signatures

Implementation /1

- Permuting rows even once is very expensive
- Approximate permutation by a hash function h_i
  - $h_i(x) = [((a \cdot x + b) \bmod p) \bmod N] + 1$
  - a, b: random integers;
    p: a prime ($p > N$); N: #rows in the matrix
  - h_i is possibly not injective, but errors are rare => OK
- Pick about **K = 100** such hash functions h_i



Implementation /2

- Pick about **K = 100** such hash functions h_i

Intuition: for fixed C and h_i, find the smallest value $h_i(r)$ over all rows r with C(r) = 1

<u>One-pass implementation</u>:

- For each column ***C*** and hash-function h_i prepare a "slot" (variable) ***sig(C)[i]*** for the min-hash value
- Initialize all ***sig(C)[i]*** = ∞
- **Scan rows of *C* looking for 1s,** with ***q*** = row index
  - If row ***q*** has 1 in column ***C***, then for each h_i (i=1..100):
    - If $h_i(q) < sig(C)[i]$, then $sig(C)[i] \leftarrow h_i(q)$

Implementation /3

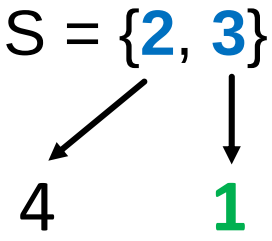
For fixed C and h_i:

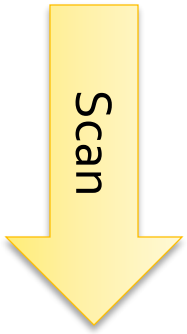
Find the smallest value $h_i(r)$ over all rows r with C(r) = 1

Scan rows looking for 1s

- If row ***q*** has 1 in column ***C***, then for each h_i :
  - If $h_i(q) < sig(C)[i]$, then $sig(C)[i] \leftarrow h_i(q)$

Example: fixed ***C*** and h_i





	C	
1	0	$h_i(1) = 3$
2	1	$h_i(2) = 4$
3	1	$h_i(3) = 1$
4	0	$h_i(4) = 2$

sig(C)[i]

sig(C)[i]
∞
4
1
1

Thank you.

Questions?

MH-Property: Detailed Proof

0	0
0	0
1	**1**
0	0
0	1
1	0

- **Detailed proof:**
  - Let **X** be a set (of shingles), ***y*** $\in$ ***X*** an element
  - **Then: Pr[π(y) = min(π(X))] = 1/|X|**
    - It is equally likely that any ***y*** $\in$ ***X*** is mapped to the ***min***
  - Let ***y*** be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - **Then either:** $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, **or**

    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$

    One of the two cols had to have 1 at position ***y***
  - So the prob. that **both** are true is the prob. ***y*** $\in C_1 \cap C_2$
  - **Pr[min($\pi(C_1)$)=min($\pi(C_2)$)]=$|C_1 \cap C_2| / |C_1 \cup C_2|$**
    = *sim*(C_1, C_2)