MATH 34B INTEGRATION WORKSHEET SOLUTIONS

* indicates that there was a typo in the original worksheet.

1. $\int \sqrt{\pi} dx =$ (hint: $\sqrt{\pi}$ is just a number.)

Solution.

$$\int \sqrt{\pi} dx = \sqrt{\pi} \int dx = \sqrt{\pi} x + C.$$

*2. $\int \frac{3}{x^8} + \frac{e}{\sqrt[8]{x^3}} dx =$

Solution.

$$\begin{aligned} \int \frac{3}{x^8} + \frac{e}{\sqrt[8]{x^3}} dx &= \int 3x^{-8} + \frac{e}{x^{\frac{3}{8}}} dx \\ &= 3 \int x^{-8} dx + e \int x^{\frac{-3}{8}} dx \\ &= 3(\frac{x^{-8+1}}{-7}) + e(\frac{x^{\frac{-3}{8}+1}}{\frac{-3}{8}+1}) + C \\ &= \frac{3}{-7} x^{-7} + e(\frac{x^{\frac{5}{8}}}{\frac{5}{8}}) + C \\ &= \frac{-3}{7} x^{-7} + \frac{8e}{5} x^{\frac{5}{8}} + C. \end{aligned}$$

3. $\int \frac{3x^2-4x+8}{x^5} dx =$ (hint: split up the fraction first.)

Solution.

$$\begin{aligned} \int \frac{3x^2-4x+8}{x^5} dx &= \int \frac{3}{x^3} - \frac{4}{x^4} + \frac{8}{x^5} dx \\ &= 3 \int x^{-3} dx - 4 \int x^{-4} dx + 8 \int x^{-5} dx \\ &= 3(\frac{x^{-3+1}}{-3+1}) - 4(\frac{x^{-4+1}}{-4+1}) + 8(\frac{x^{-5+1}}{-5+1}) + C \\ &= 3(\frac{x^{-2}}{-2}) - 4(\frac{x^{-3}}{-3}) + 8(\frac{x^{-4}}{-4}) + C \\ &= \frac{-3}{2} x^{-2} + \frac{4}{3} x^{-3} - 2x^{-4} + C. \end{aligned}$$

4. $\int (3x+1)(x+2)^2 dx =$ (hint: distribute/foil first.)

Solution.

$$\begin{aligned}
\int (3x+1)(x+2)^2 dx &= \int (3x+1)(x^2+4x+4)dx \\
&= \int (3x^3+12x^2+12x+x^2+4x+4)dx \\
&= \int (3x^3+13x^2+16x+4)dx \\
&= 3\int x^3 dx + 13\int x^2 dx + 16\int x dx + 4\int dx \\
&= 3(\frac{x^4}{4}) + 13(\frac{x^3}{3}) + 16(\frac{x^2}{2}) + 4x + C \\
&= \frac{3}{4}x^4 + \frac{13}{3}x^3 + 8x^2 + 4x + C.
\end{aligned}$$

5. $\int \ln(2e^{\sin(x)})dx =$ (hint: use log rules to simplify this first; there are two rules involved.)

Solution.

$$\begin{aligned}
\int \ln(2e^{\sin x})dx &= \int (\ln 2 + \ln e^{\sin x})dx \\
&= \int \ln 2 dx + \int \sin x dx \\
&= (\ln 2)x - \cos x + C.
\end{aligned}$$

(Use u-substitution for the rest of the problems.)

*6. $\int 2y^2 e^{\pi - y^3} dy =$

Solution. Let $u = \pi - y^3$. Then, $du = -3y^2 dy \implies \frac{du}{-3} = y^2 dy$ so the integral becomes

$$\begin{aligned}
\int 2y^2 e^{\pi-y^3} dy &= \int 2(e^{\pi-y^3})(y^2 dy) \\
&= 2\int e^u(\frac{du}{-3}) \\
&= \frac{-2}{3}\int e^u du \\
&= \frac{-2}{3}e^u + C \\
&= \frac{-2}{3}e^{\pi-y^3} + C.
\end{aligned}$$

*7. $\int \frac{t^2+2t}{\sqrt[7]{t^3+3t^2+10}}dt =$

Solution. Let $u = t^3 + 3t^2 + 10$. Then, $du = (3t^2 + 6t)dt = 3(t^2 + 2t)dt \implies \frac{du}{3} = (t^2 + 2t)dt$. So,

$$\begin{aligned} \int \frac{t^2 + 2t}{\sqrt[7]{t^3 + 3t^2 + 10}} dt &= \int \frac{\frac{du}{3}}{\sqrt[7]{u}} \\ &= \frac{1}{3} \int u^{\frac{1}{7}} du \\ &= \frac{1}{3} \left(\frac{u^{\frac{1}{7}+1}}{\frac{1}{7} + 1} \right) + C \\ &= \frac{1}{3} \left(\frac{u^{\frac{8}{7}}}{\frac{8}{7}} \right) + C \\ &= \frac{7}{24} (t^3 + 3t^2 + 10)^{\frac{8}{7}} + C. \end{aligned}$$

8. $\int \frac{x}{3x^2+8} dx =$

Solution. Let $u = 3x^2 + 8$. Then $du = 6xdx$ and so $\frac{du}{6} = xdx$. Hence,

$$\begin{aligned} \int \frac{x}{3x^2 + 8} dx &= \int \frac{\frac{du}{6}}{u} \\ &= \frac{1}{6} \int \frac{1}{u} du \\ &= \frac{1}{6} \ln u + C \\ &= \frac{1}{6} \ln(3x^2 + 8) + C. \end{aligned}$$

*9. $\int \frac{\cos x}{\sin^2 x} dx =$ (hint: this is similar to the previous problem.)

Solution. Let $u = \sin x$. Then $du = \cos x dx$ and so the integral becomes

$$\begin{aligned} \int \frac{\cos x}{\sin^2 x} dx &= \int \frac{du}{u^2} \\ &= \int u^{-2} du \\ &= \frac{u^{-2+1}}{-2 + 1} + C \\ &= -u^{-1} + C \\ &= -(\sin x)^{-1} + C. \end{aligned}$$

10. $\int \frac{4}{x \ln x} dx =$

Solution. Let $u = \ln x$. Then $du = \frac{1}{x} dx$ and so

$$\begin{aligned} \int \frac{4}{x \ln x} dx &= 4 \int \frac{1}{\ln x} \left(\frac{dx}{x} \right) \\ &= 4 \int \frac{1}{u} du \\ &= 4 \ln u + C \\ &= 4 \ln(\ln x) + C. \end{aligned}$$

*11. $\int (\sin^2 x + 1)(\cos x + 2) dx =$ (hint: distribute; double angle formula; u-sub)

Solution. First we distribute.

$$\begin{aligned}\int(\sin^2 x+1)(\cos x+2)dx &= \int \sin^2 x\cos x + 2\sin^2 x+\cos x+2dx\\ &= \int \sin^2 x\cos x dx + 2\int \sin^2 x dx + \int \cos x dx + 2\int dx.\end{aligned}$$

Now we integrate each integral separately. The last two are easy.

$$2\int dx = 2x + C_1.$$

$$\int \cos x dx = \sin x + C_2.$$

For the first integral, we use u-sub with $u=\sin x$. Then $du=\cos x dx$ and we get

$$\int \sin^2 x\cos x dx = \int u^2 du = \frac{u^3}{3}+C_3 = \frac{\sin^3 x}{3}+C_3.$$

For the second integral, we use the double angle formula.

$$\begin{aligned}2\int \sin^2 x dx &= 2\int \frac{1-\cos 2x}{2}dx\\ &= \int (1-\cos 2x)dx\\ &= \int dx - \int \cos 2x dx\\ &= x - \frac{\sin 2x}{2} + C_4.\end{aligned}$$

(You can use a u-sub to integrate $\cos 2x$ also; but it's easier if you just think backwards.) Notice that we used different C_i's for each integral because they are different constants. But when we add everything up at the end, we can combine them all together to become one single constant. Hence,

$$\int \sin^2 x\cos x dx + 2\int \sin^2 x dx + \int \cos x dx + 2\int dx = (\frac{\sin^3 x}{3}) + (x - \frac{\sin 2x}{2}) + (\sin x) + (2x) + C$$

is the final answer.