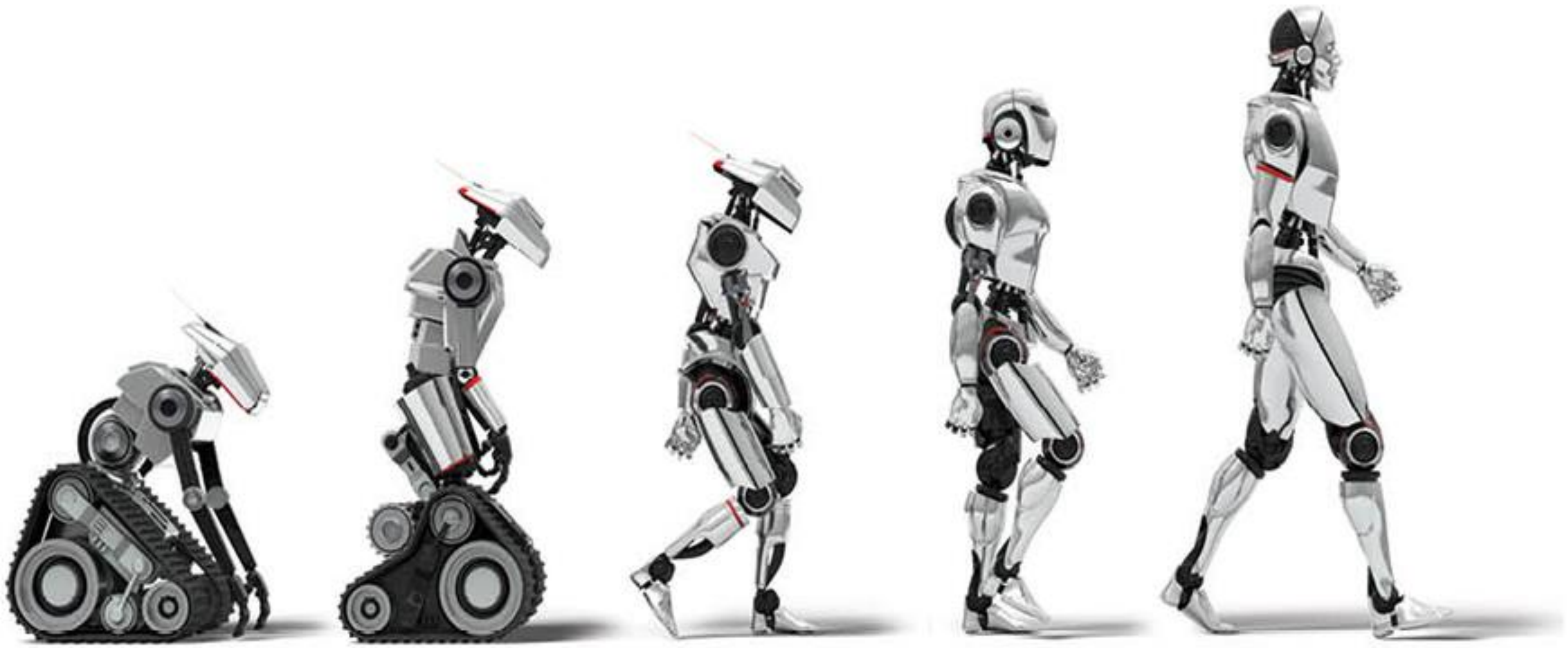
UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Universität Heidelberg

Fakultät für Physik und Astronomie

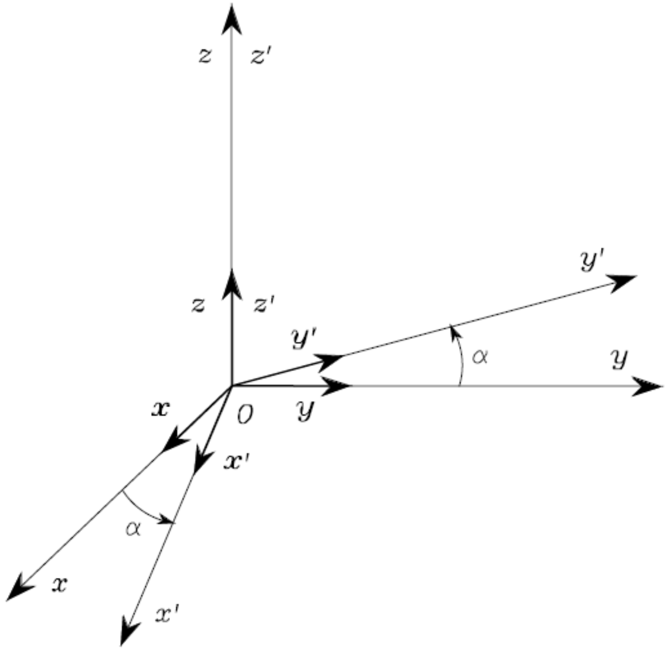
Exercises Robotics 2



Rotation Matrix: Definition

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{x}' & \boldsymbol{y}' & \boldsymbol{z}' \end{bmatrix} = \begin{bmatrix} x'_x & y'_x & z'_x \\ x'_y & y'_y & z'_y \\ x'_z & y'_z & z'_z \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}'^T\boldsymbol{x} & \boldsymbol{y}'^T\boldsymbol{x} & \boldsymbol{z}'^T\boldsymbol{x} \\ \boldsymbol{x}'^T\boldsymbol{y} & \boldsymbol{y}'^T\boldsymbol{y} & \boldsymbol{z}'^T\boldsymbol{y} \\ \boldsymbol{x}'^T\boldsymbol{z} & \boldsymbol{y}'^T\boldsymbol{z} & \boldsymbol{z}'^T\boldsymbol{z} \end{bmatrix}$$

R is an *orthogonal* matrix meaning that $\boldsymbol{R}^T\boldsymbol{R} = \boldsymbol{I}_3$ or $\boldsymbol{R}^T = \boldsymbol{R}^{-1}$



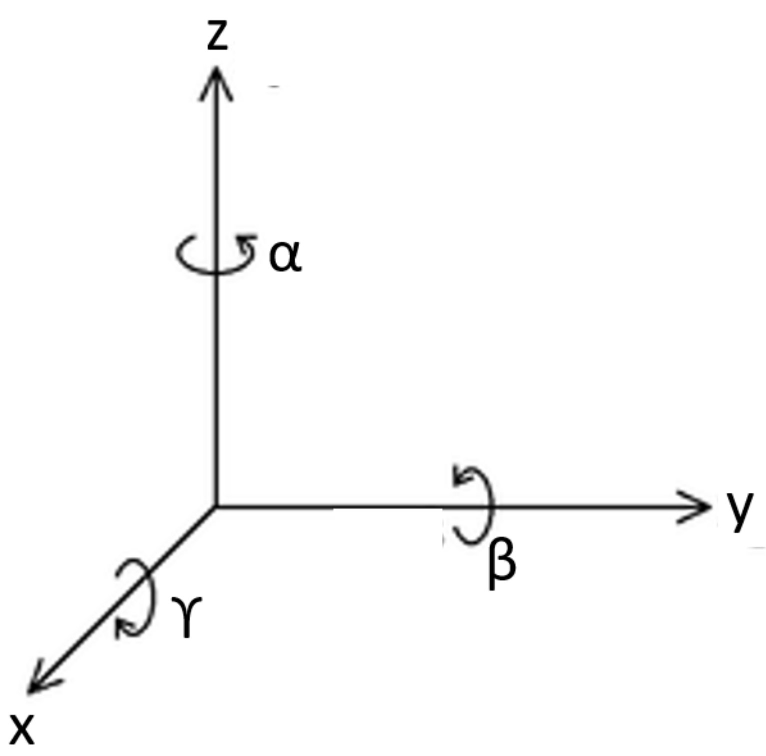
$$\boldsymbol{x}' = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix} \qquad \boldsymbol{y}' = \begin{bmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix} \qquad \boldsymbol{z}' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Rotation Matrix: Basic Rotations

$$\boldsymbol{R}_z(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$\boldsymbol{R}_x(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$



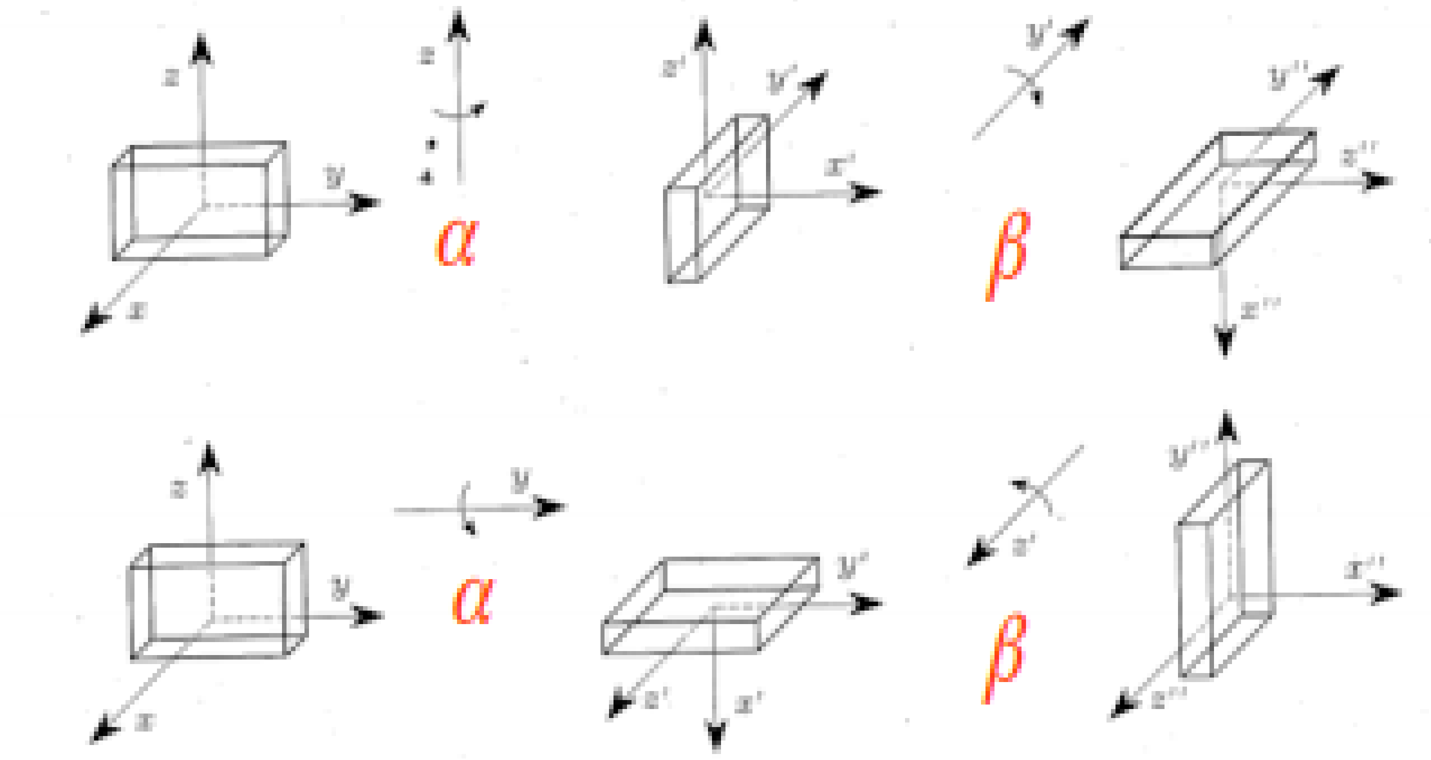
Rotation Matrix

Consider the following matrix

$$^{A}\boldsymbol{R}_B(\rho,\sigma) = \begin{pmatrix} \cos\rho & -\sin\rho & 0 \\ \sin\rho\cos\sigma & \cos\rho\cos\sigma & -\sin\sigma \\ \sin\rho\sin\sigma & \cos\rho\sin\sigma & \cos\sigma \end{pmatrix}.$$

- Prove that this is a rotation matrix (representing thus the orientation of a frame B with respect to a fixed frame A) for any value of the pair of angles (ρ,σ).
- Which is the sequence of two elementary rotations around *fixed* coordinate axes providing $^{A}\boldsymbol{R}_B(\rho,\sigma)$?
- Verify your statements for $\rho = 90°$ and $\sigma = -90°$.

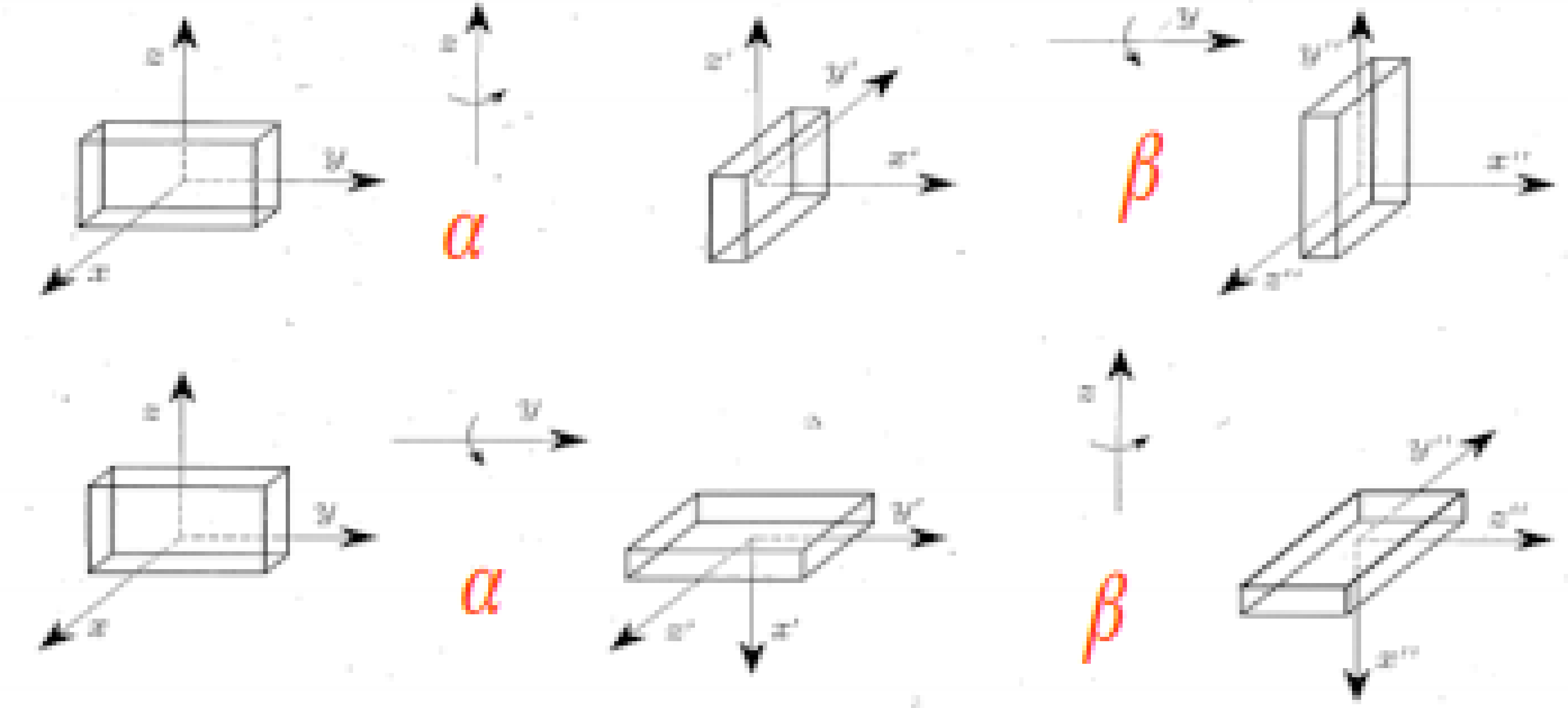
Rotation Matrix: Mobile Frame



$$R_j = R_{Z,\alpha} R_{y,\beta}$$

$$R_j = R_{Y,\alpha} R_{z,\beta}$$

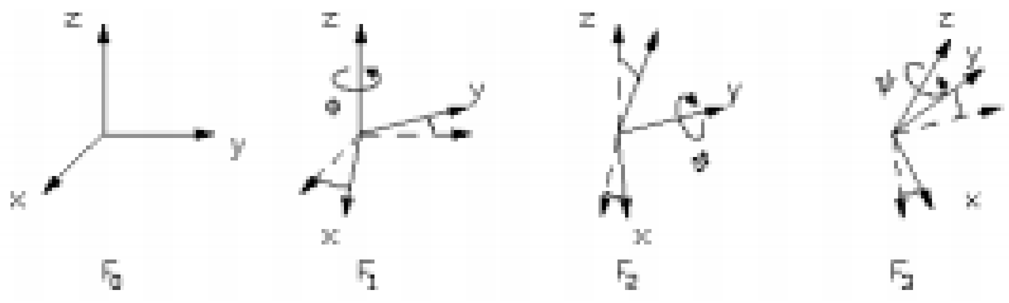
Rotation Matrix: Fixed Frame



$$R_j = R_{Y,\beta} R_{Z,\alpha}$$

$$R_j = R_{Z,\beta} R_{Y,\alpha}$$

Rotation Matrix: Mobile Frame



Considering the following rotations:

- FF1 around z
- FF2 around y'
- FF3 around z''

Starting from the fixed frame FF0 solve the foward and inverse problem of the given Euler angles.

Grübler's equation

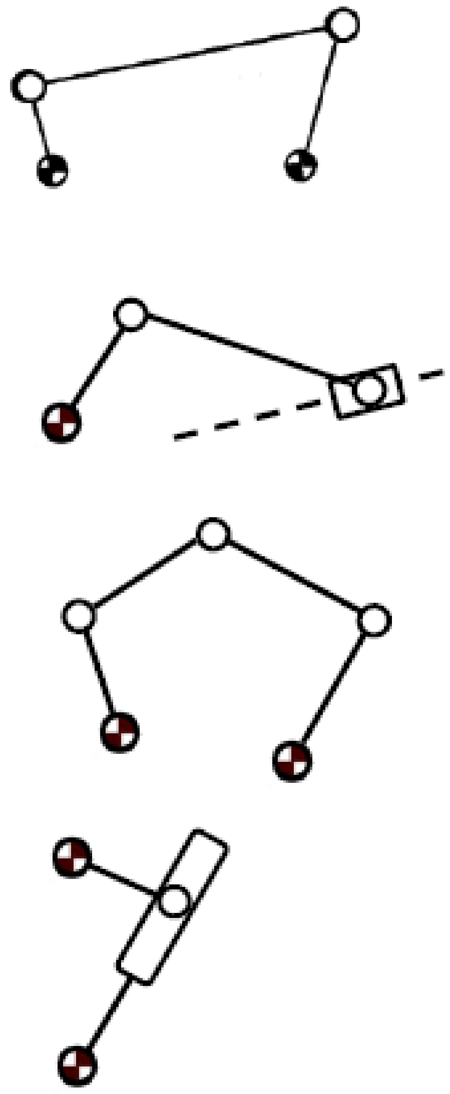
$$F = \lambda(n-1) - \sum_{i=1}^{j} c_i$$ **Grübler Formula**

λ DOFs of the operating space of the mechanism (3 if on the plane...6 in space)

n Number of links

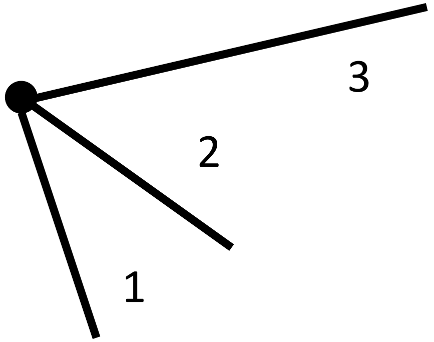
j Number of kinematic pairs

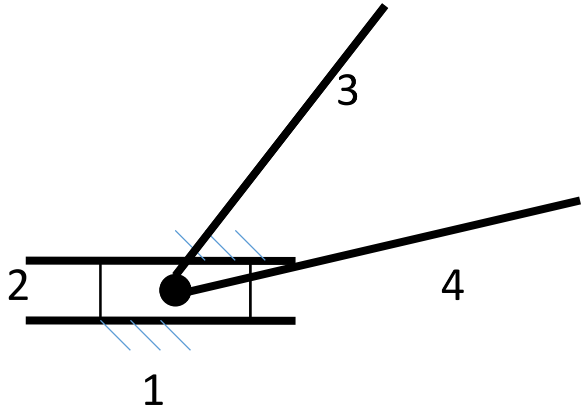
c_i Degree of constraint of the i-th kinematic pair



Grübler's equation

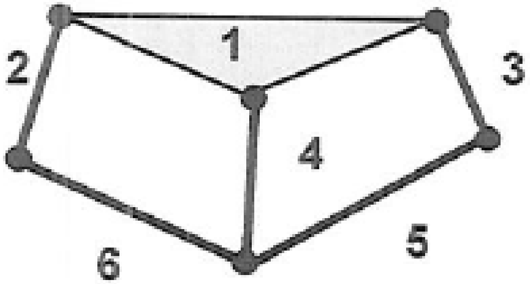
Some mechanism have three kinematic pairs concurring on the same joint

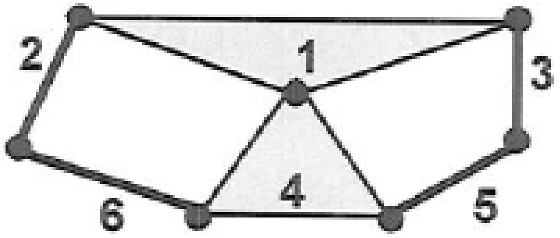




Such configuration can lead to confusion for mechanical modeling

Grübler’s equation





$$F = 3(n-1) - 2J_L - J_H = 1$$

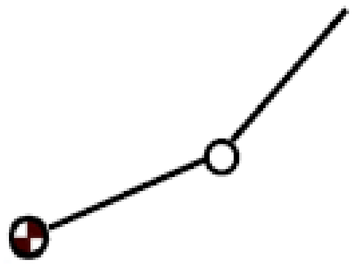
$$J_H = 0$$

$$J_L = 7$$

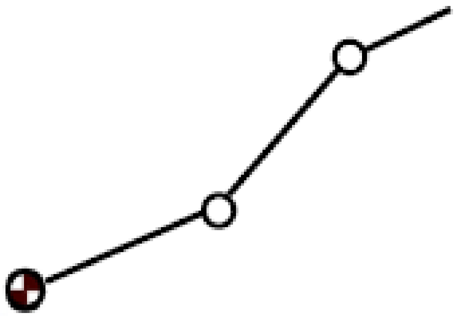
$$n = 6$$

In this condition the mechanism can be decomposed and applying the Grübler Formula would allow to find the exact number of degrees of freedom

Grübler's equation



$$m = 3 \qquad c_1 = 2$$
$$n = (3-1)\cdot 3 - 2\cdot 2 = 2$$

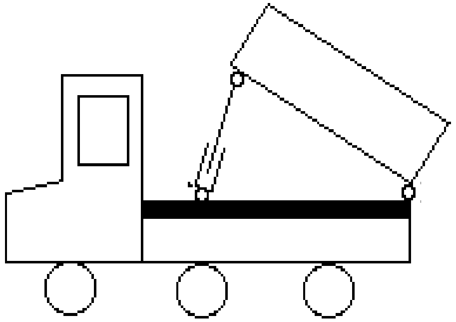


$$m = 4 \qquad c_1 = 3$$
$$n = (4-1)\cdot 3 - 2\cdot 3 = 3$$

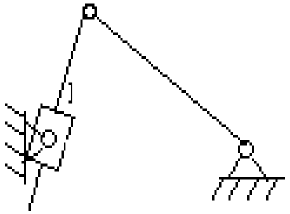
IMPORTANT: with open loop mechanisms the number of DOF can be calculated also as the sum of the DOF allowed by the cinematic joints.

DOF=C1+2C2

Grübler’s equation

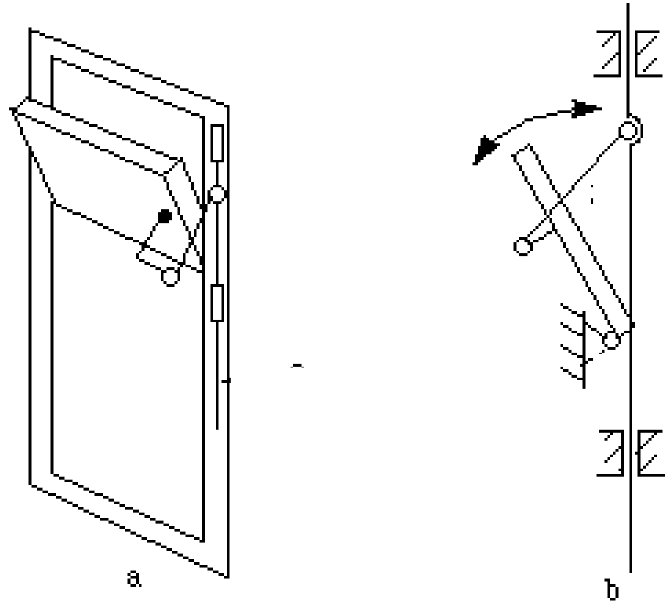


a



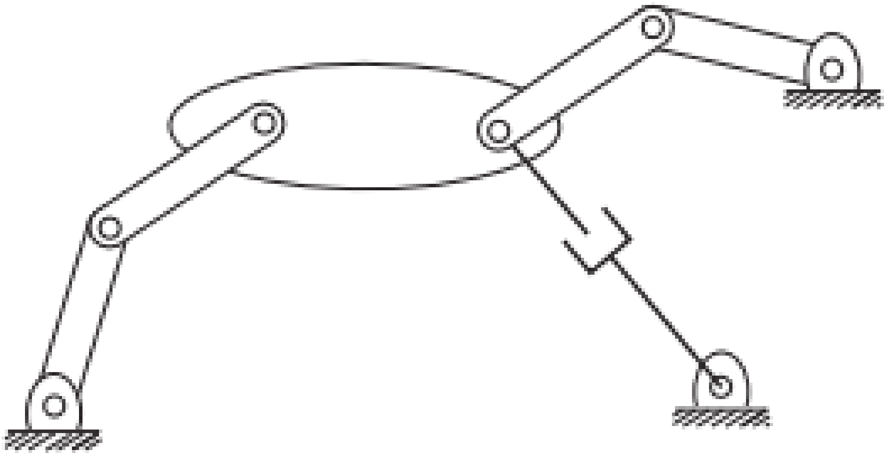
b

Grübler’s equation



Grübler’s equation

PLANAR MECHANISM WITH OVERLAPPING JOINTS:



Thank you for your Attention!!!