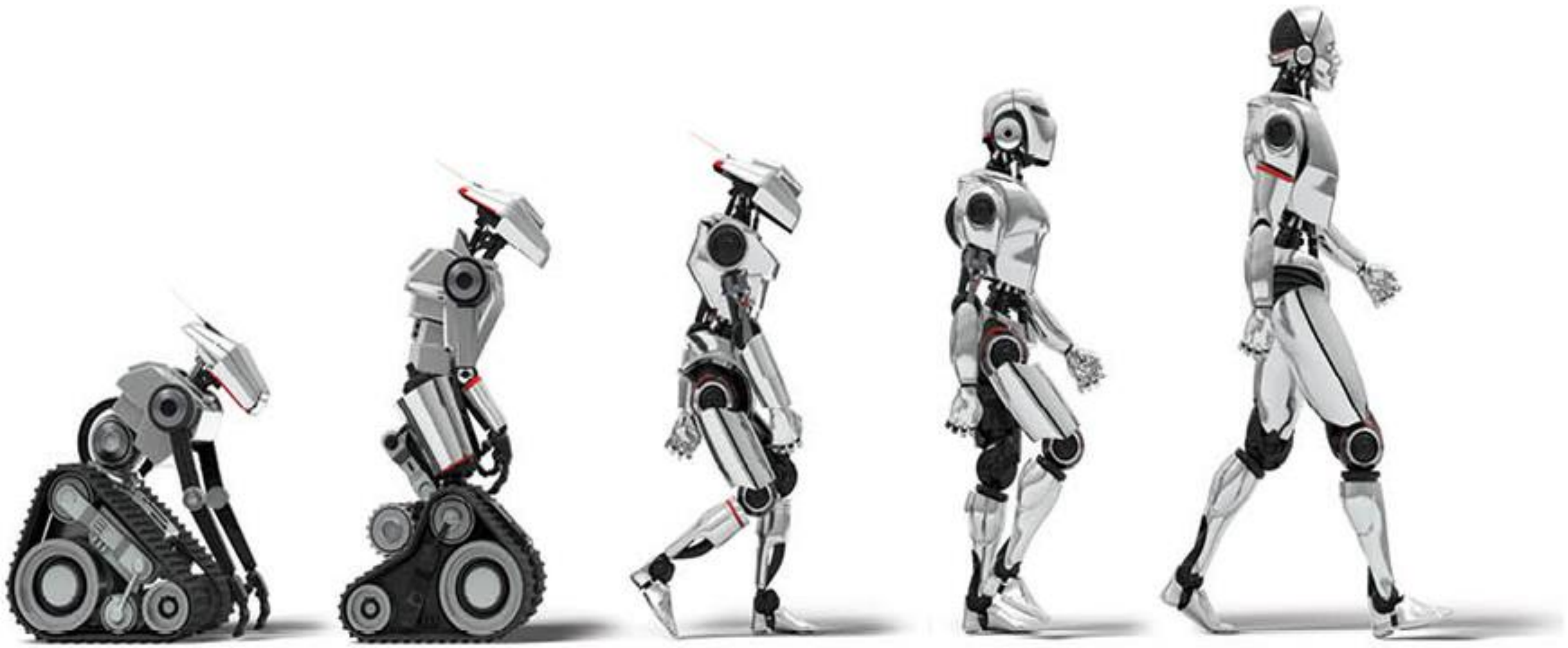
UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Universität Heidelberg

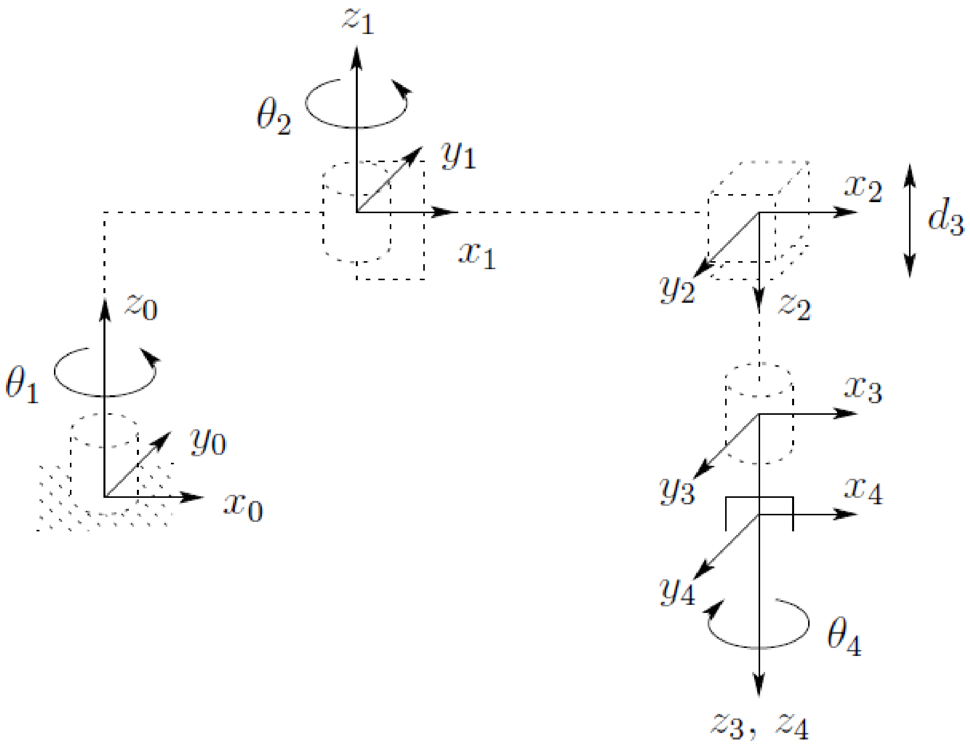
Fakultät für Physik und Astronomie

Exercises Robotics 5

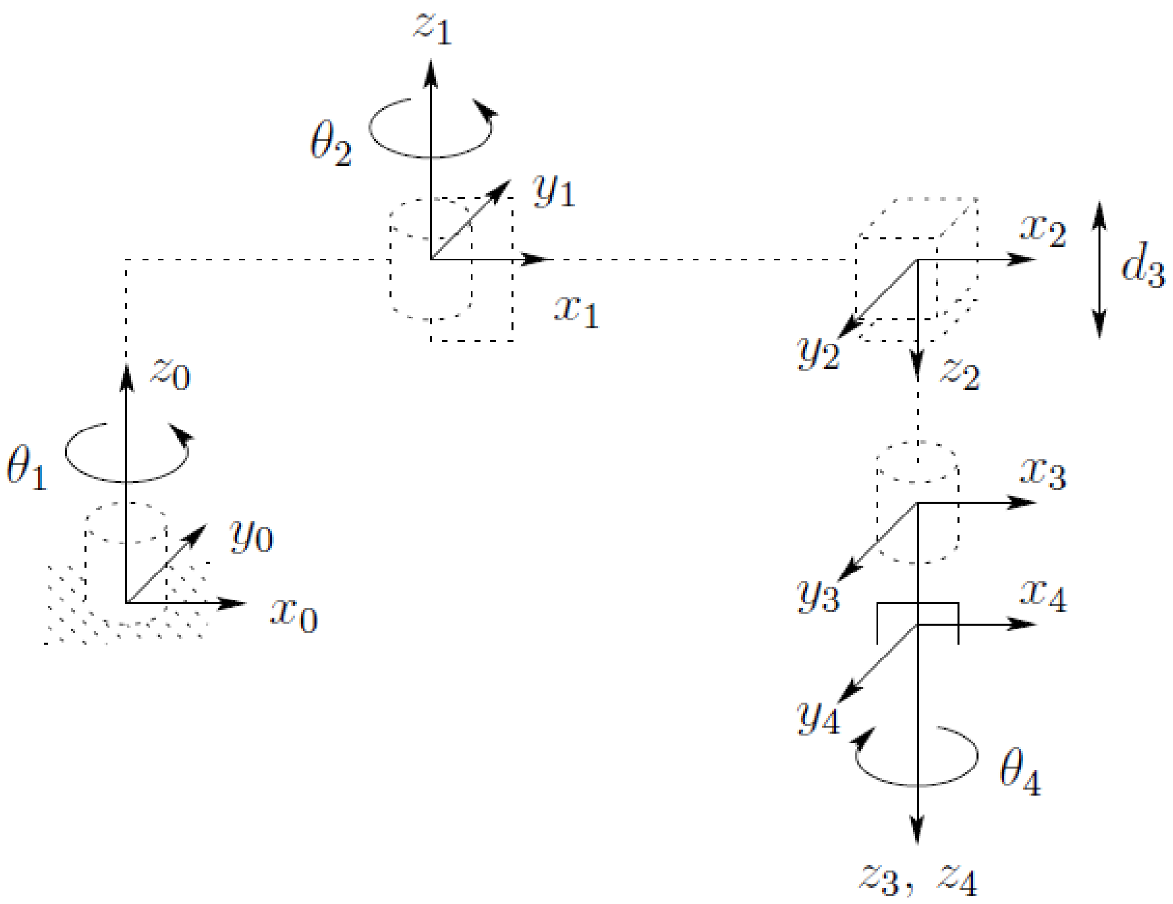


Exercises D-H: SCARA Manipulator

As another example of the general procedure, consider the SCARA manipulator.
This manipulator, consists of an RRP arm and a one degree-of-freedom wrist, whose motion is a roll about the vertical axis.



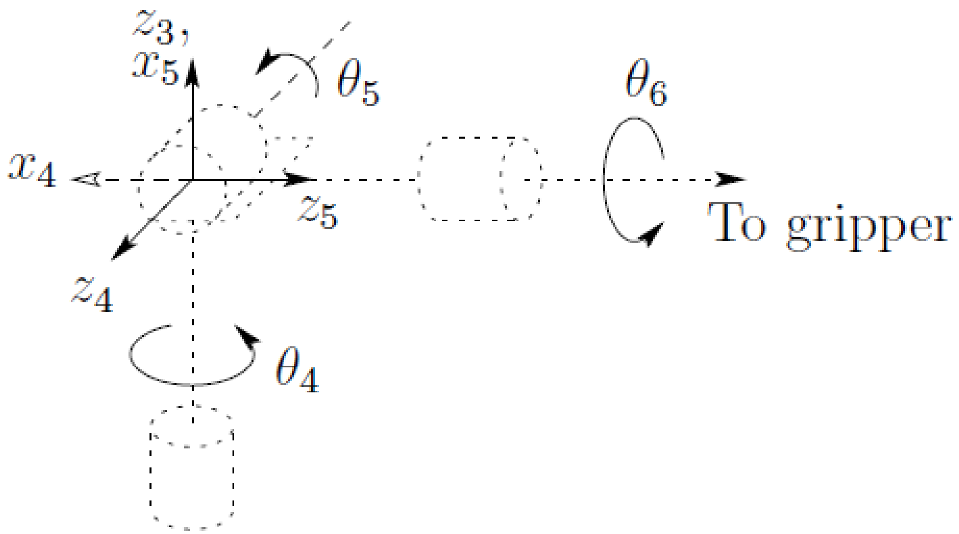
Exercises Denavit-Hartenberg



Variable coordinates are in RED

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	180	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

Inverse Kinematics: Spherical Wirst



Link	a_i	α_i	d_i	θ_i
4	0	−90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

* variable

$$T_6^3 = A_4 A_5 A_6 = \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordinates of the end-effector respect to the base (in this case is link 3 the base which is not visible)

Rotation of the end-effector around the frame x4 y4 z4

Inverse Kinematics: Spherical Wirst

kinematic decoupling (orientation)

From Euler Angle (lecture 3)

$$R_{ZYZ}=\begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

From Euler Angle spherical wrist (lecture 3)

$$T_6^3 = A_4 A_5 A_6 = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -s_4 c_6 - c_4 c_5 s_6 & c_4 s_5 & d_6^* c_4 s_5 \\ c_5 c_6 s_4 + c_4 s_6 & c_4 c_6 - c_5 s_4 s_6 & s_4 s_5 & d_6^* s_4 s_5 \\ -c_6 s_5 & s_5 s_6 & c_5 & d_6^* c_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let $\phi = \theta_4^*, \theta = \theta_5^*$, and $\psi = \theta_6^*$

Inverse Kinematics: Spherical Wirst

$$\boldsymbol{R}(\boldsymbol{\phi}) = \boldsymbol{R}_z(\varphi)\boldsymbol{R}_{y'}(\vartheta)\boldsymbol{R}_{z''}(\psi) = \begin{bmatrix} c_\varphi c_\vartheta c_\psi - s_\varphi s_\psi & -c_\varphi c_\vartheta s_\psi - s_\varphi c_\psi & c_\varphi s_\vartheta \\ s_\varphi c_\vartheta c_\psi + c_\varphi s_\psi & -s_\varphi c_\vartheta s_\psi + c_\varphi c_\psi & s_\varphi s_\vartheta \\ -s_\vartheta c_\psi & s_\vartheta s_\psi & c_\vartheta \end{bmatrix}$$

It is useful to solve the *inverse problem*, that is to determine the **set of Euler** angles corresponding to a given rotation matrix (known)

$$\boldsymbol{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

By considering the elements [1, 3] and [2, 3] $\varphi = \mathrm{Atan2}(r_{23}, r_{13})$

Inverse Kinematics: Spherical Wirst

Then, squaring and summing the elements [1, 3] and [2, 3] and using the element [3, 3] yields

$$\vartheta = \mathrm{Atan2}\left(\sqrt{r_{13}^2 + r_{23}^2}, r_{33}\right)$$

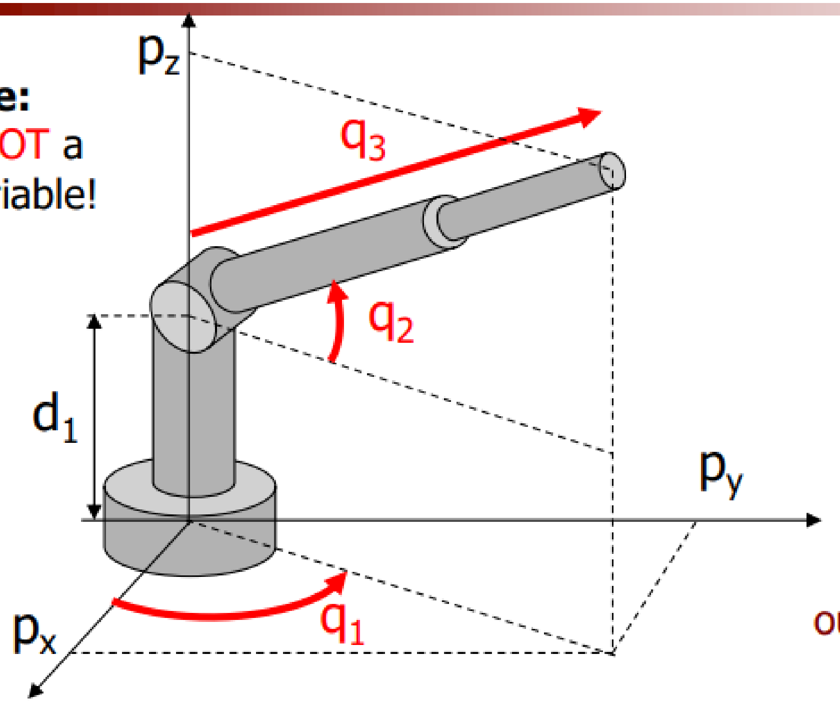
The choice of the positive sign for the term $r^2_{13} + r^2_{23}$ limits the range of feasible values of ϑ to $(0, \pi)$.

On this assumption, considering the elements [3, 1] and [3, 2] gives

$$\psi = \mathrm{Atan2}(r_{32}, -r_{31})$$

Inverse Kinematics: RPR Arm

Note:
q_2 is NOT a DH variable!



$$p_x = q_3\, c_2\, c_1$$

$$p_y = q_3\, c_2\, s_1$$

$$p_z = d_1 + q_3\, s_2$$

$$p_x^2 + p_y^2 + (p_z - d_1)^2 = q_3^2$$

$$q_3 = + \sqrt{p_x^2 + p_y^2 + (p_z - d_1)^2}$$

our choice: take here only the positive value...

if $q_3 = 0$, then q_1 and q_2 remain both undefined (stop); else

$$q_2 = \text{ATAN2}\{(p_z - d_1)/q_3, \pm \sqrt{(p_x^2 + p_y^2)/q_3^2}\}$$

(if it stops, a singular case: ∞^2 or ∞^1 solutions)

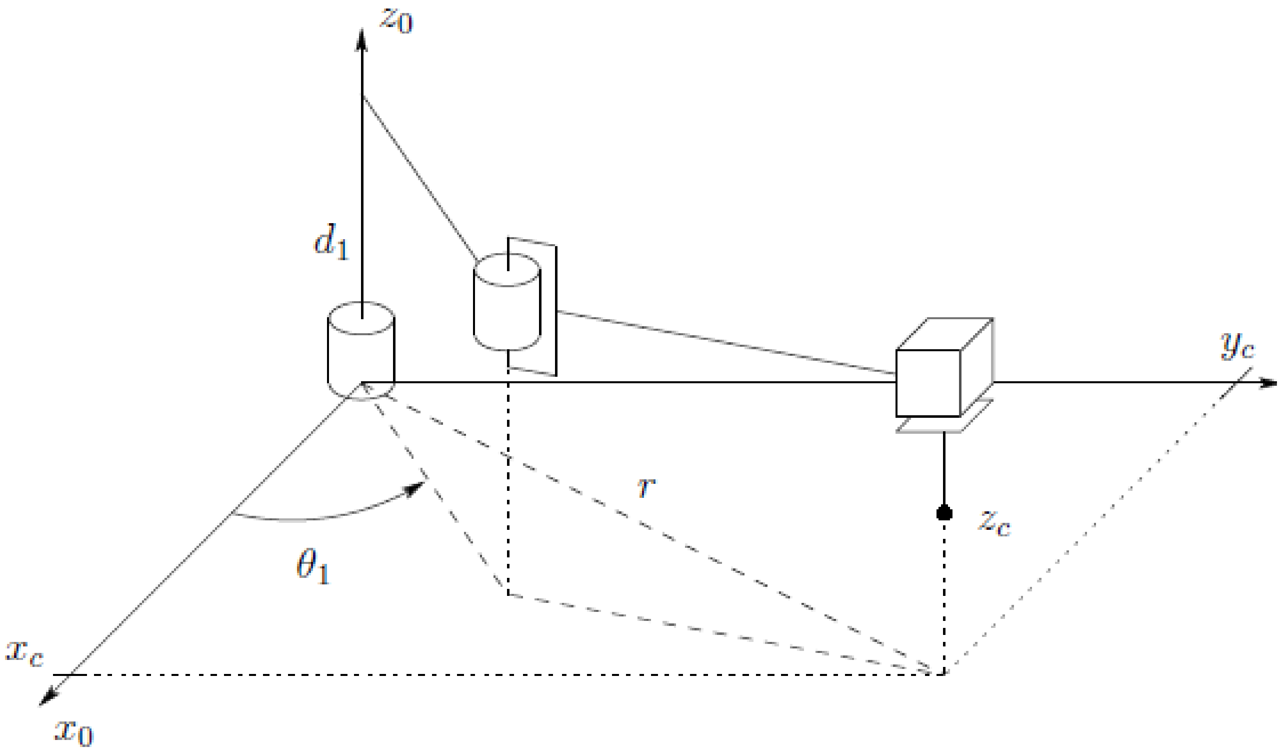
if $p_x^2+p_y^2 = 0$, then q_1 remains undefined (stop); else

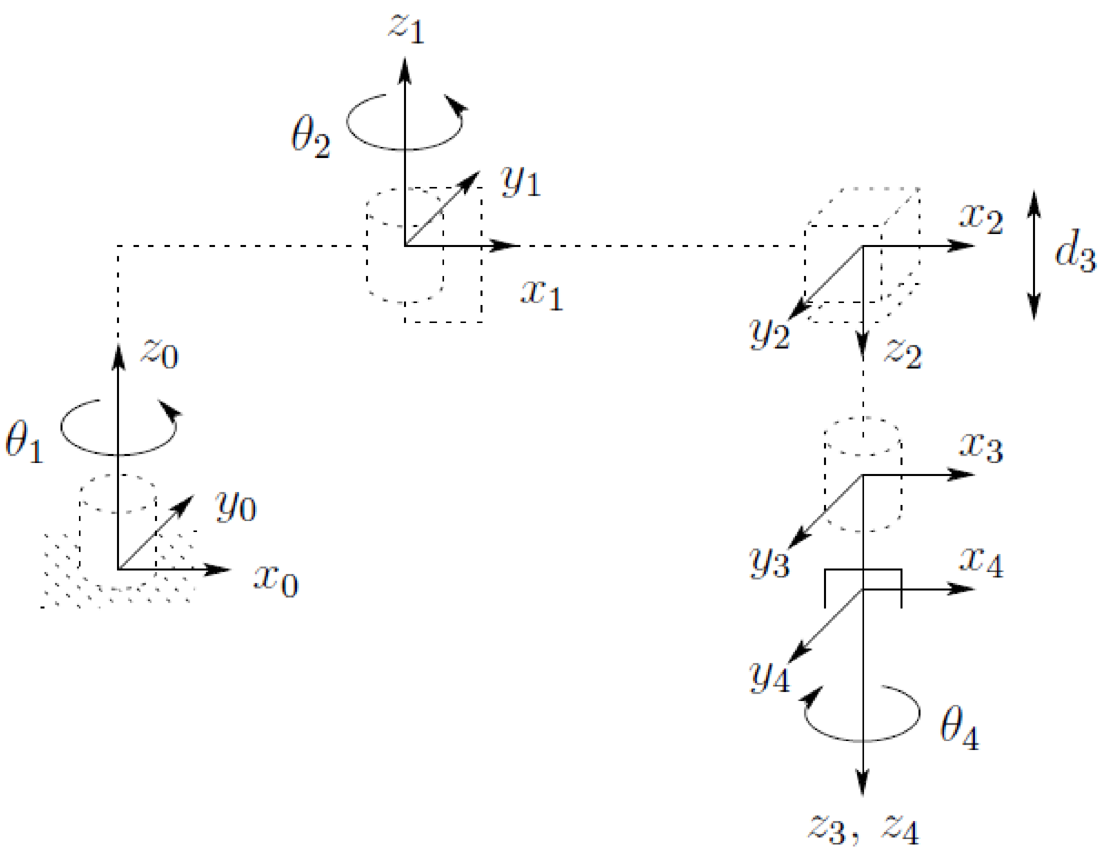
$$q_1 = \text{ATAN2}\{p_y/c_2, p_x/c_2\}$$

(2 regular solutions $\{q_1,q_2,q_3\}$)

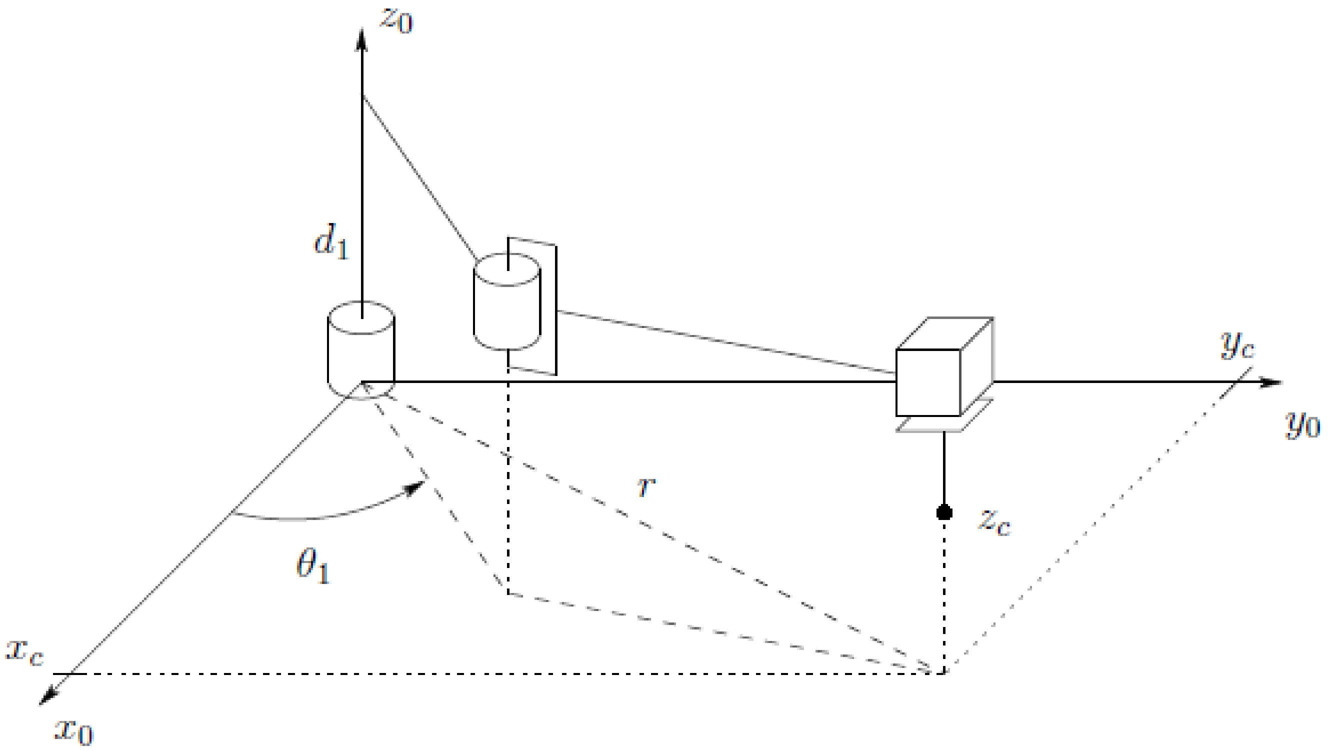
we have eliminated $q_3>0$ from both arguments!

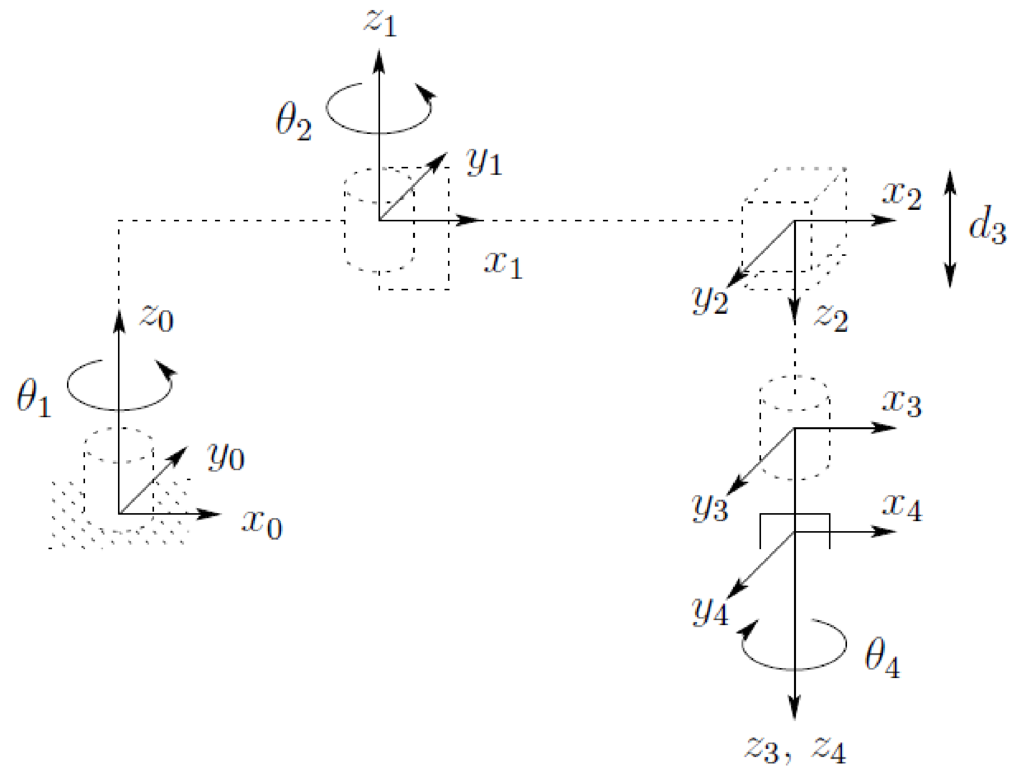
Inverse Kinematics: SCARA Manupulator





Inverse Kinematics: SCARA Manupulator





$$T_4^0 = A_1 \cdots A_4 \quad = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	180	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

Inverse Kinematics: SCARA Manipulator

The transformation from the base 0 to the end effector 4 is a rotation matrix given by:

$$R = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ s_\alpha & -c_\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \theta_1 + \theta_2 - \theta_4 = \alpha = A\tan(r_{11}, r_{12})$$

$$\textit{We see from this that} \quad \theta_2 = A\tan\left(c_2, \pm\sqrt{1 - c_2}\right)$$

$$\textit{where} \quad c_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$\theta_1 = A\tan(o_x, o_y) - A\tan(a_1 + a_2c_2, a_2s_2).$$

We may then determine θ_4 *from*

$$\theta_4 = \theta_1 + \theta_2 - \alpha = \theta_1 + \theta_2 - A\tan(r_{11}, r_{12}).$$

$$\textit{Finally } d_3 \textit{ is given as} \quad d_3 = o_z + d_4.$$

Inverse Kinematics: SCARA Manipulator

To find Θ_2: if we square and sum P_x and P_y, we can get an expression in Θ_2:

$$P_x^2 + P_y^2 = (a_1 C_1 + a_2 C_{1-2})^2 + (a_1 S_1 + a_2 S_{1-2})^2$$

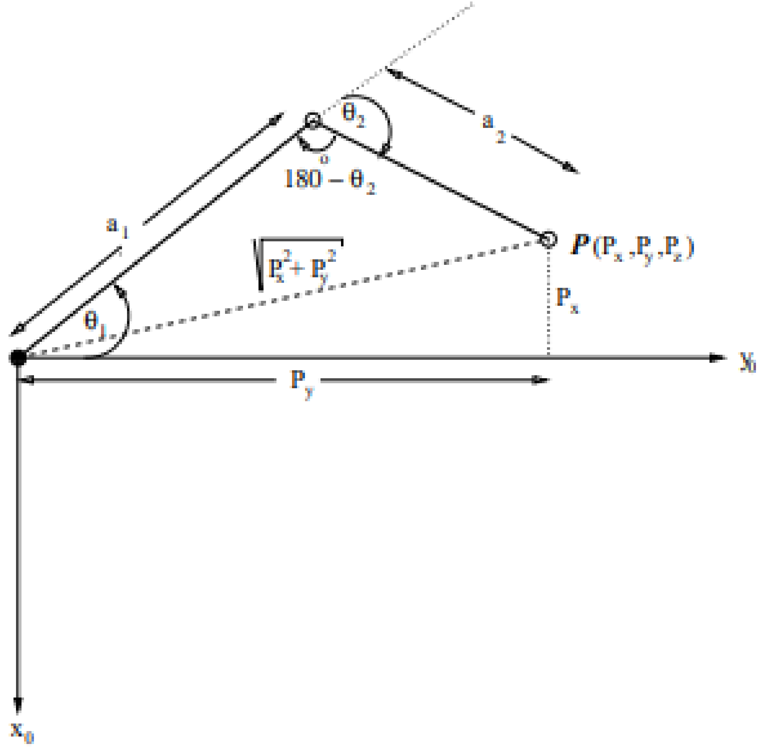
$$P_x^2 + P_y^2 = a_1^2 + a_2^2 + 2a_1 a_2 C_1 (C_1 C_2 + S_1 S_2) + 2a_1 a_2 S_1 (S_1 C_2 - S_2 C_1)$$

$$P_x^2 + P_y^2 = a_1^2 + a_2^2 + 2a_1 a_2 C_1^2 C_2 + 2a_1 a_2 S_1^2 C_2$$

$$P_x^2 + P_y^2 = a_1^2 + a_2^2 + 2a_1 a_2 C_2$$

$$C_2 = \frac{P_x^2 + P_y^2 - a_1^2 - a_2^2}{2a_1 a_2} \ ; \ S_2 = \sqrt{1 - C_2^2}$$

$$\Theta_2 = \pm\, Cos^{-1}\left(\frac{P_x^2 + P_y^2 - a_1^2 - a_2^2}{2a_1 a_2}\right)$$



Solution to Θ_2 of Adept, as seen from above (along Z axis)

Inverse Kinematics: SCARA Manipulator

To solve for Θ_1, we solve for the following:

$$a_1 C_1 + a_2 C_{1-2} = P_x \qquad a_1 S_1 + a_2 S_{1-2} = P_y$$

Two equations in two unknowns (C_1, S_1)
(Θ_2 *known from above*)

$$a_1 C_1 + a_2 C_1 C_2 + a_2 S_1 S_2 = P_x \;,\; a_1 S_1 + a_2 S_1 C_2 - a_2 S_2 C_1 = P_y$$

$$(a_1 + a_2 C_2) C_1 + (a_2 S_2) S_1 = P_x \;,\; (-a_2 S_2) C_1 + (a_1 + a_2 C_2) S_1 = P_y$$

$$S_1 = \frac{a_2 S_2 P_x + (a_1 + a_2 C_2) P_y}{(a_2 S_2)^2 + (a_1 + a_2 C_2)^2}$$

$$C_1 = \frac{(a_1 + a_2 C_2) P_x - a_2 S_2 P_y}{(a_2 S_2)^2 + (a_1 + a_2 C_2)^2}$$

$$\Theta_1 = atan_2(a_2 S_2 P_x + (a_1 + a_2 C_2) P_y, (a_1 + a_2 C_2) P_x - a_2 S_2 P_y)$$

Inverse Kinematics: SCARA Manipulator

To solve for q_3:

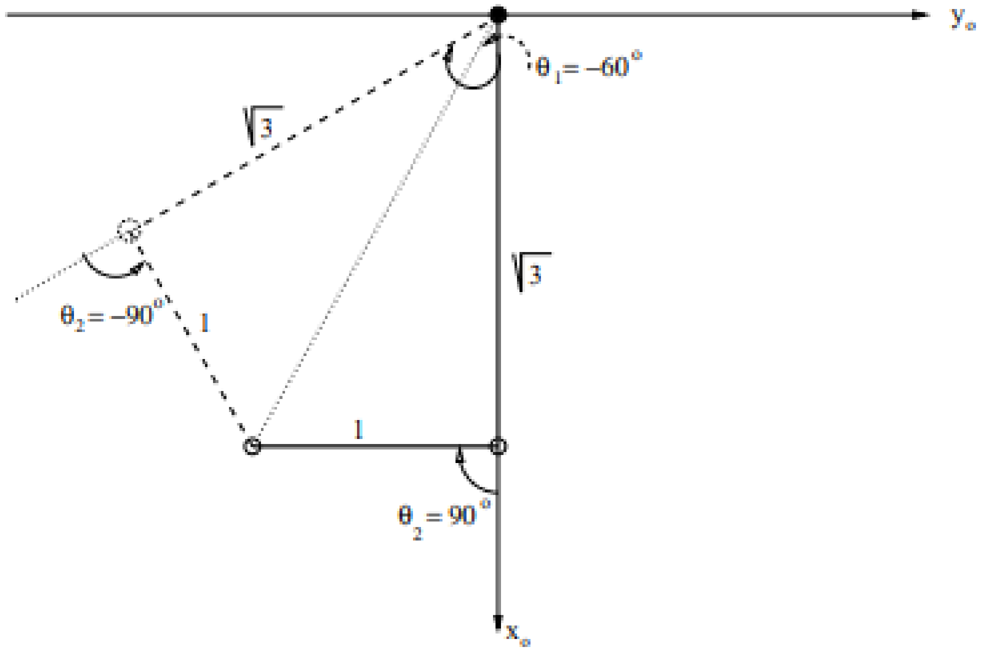
$$P_z = d_1 - q_3 - d_4 \ ; \ q_3 = d_1 - d_4 - P_z$$

To solve for Θ_4: The final roll angle cannot be determined from the position vector $[P_x, P_y, P_z]$.
If we are given the orientation matrix, then we can use the ratios of N_x, N_y to find Θ_4

$$Tan_{1-2-4} = \frac{S_{1-2-4}}{C_{1-2-4}} = \frac{N_y}{N_x}$$

$$\Theta_1 - \Theta_2 - \Theta_4 = atan_2(N_y, N_x)$$

$$\Theta_4 = -atan_2(N_y, N_x) + \Theta_1 - \Theta_2$$



Example solution ignoring Θ_4 with 2 arm positions

Inverse Kinematics: SCARA Manipulator

Example Solution (no Θ_4) if $a_1 = \sqrt{3},\ a_2 = 1,\ d_1 = 5,\ d_4 = 2$ and P=$\sqrt{3}$,-1,1], solve for joint variables:

$$\Theta_2 = \pm Cos^{-1}\left(\frac{(P_x^2 + P_y^2 - a_1^2 - a_2^2)}{2a_1a_2}\right) = \pm Cos^{-1}(\frac{0}{2\sqrt{3}}) = \pm 90^\circ$$

$$\Theta_1 = atan_2(a_2S_2P_x + (a_1 + a_2C_2)P_y, (a_1 + a_2C_2)P_x - a_2S_2P_y)$$

$$if\ \Theta_2 = +90^\circ,\ \Theta_1 = atan_2(0, 4) = 0^\circ$$

$$if\ \Theta_2 = -90^\circ,\ \Theta_1 = atan_2(-\sqrt{3}, 1) = -60^\circ$$

$$q_3 = d_1 - d_4 - P_z = 5 - 2 - 1 = 2$$

Two Solutions:

Θ_1	Θ_2	q_3
0°	90°	2
-60°	-90°	2

Thank you for your Attention!!!