

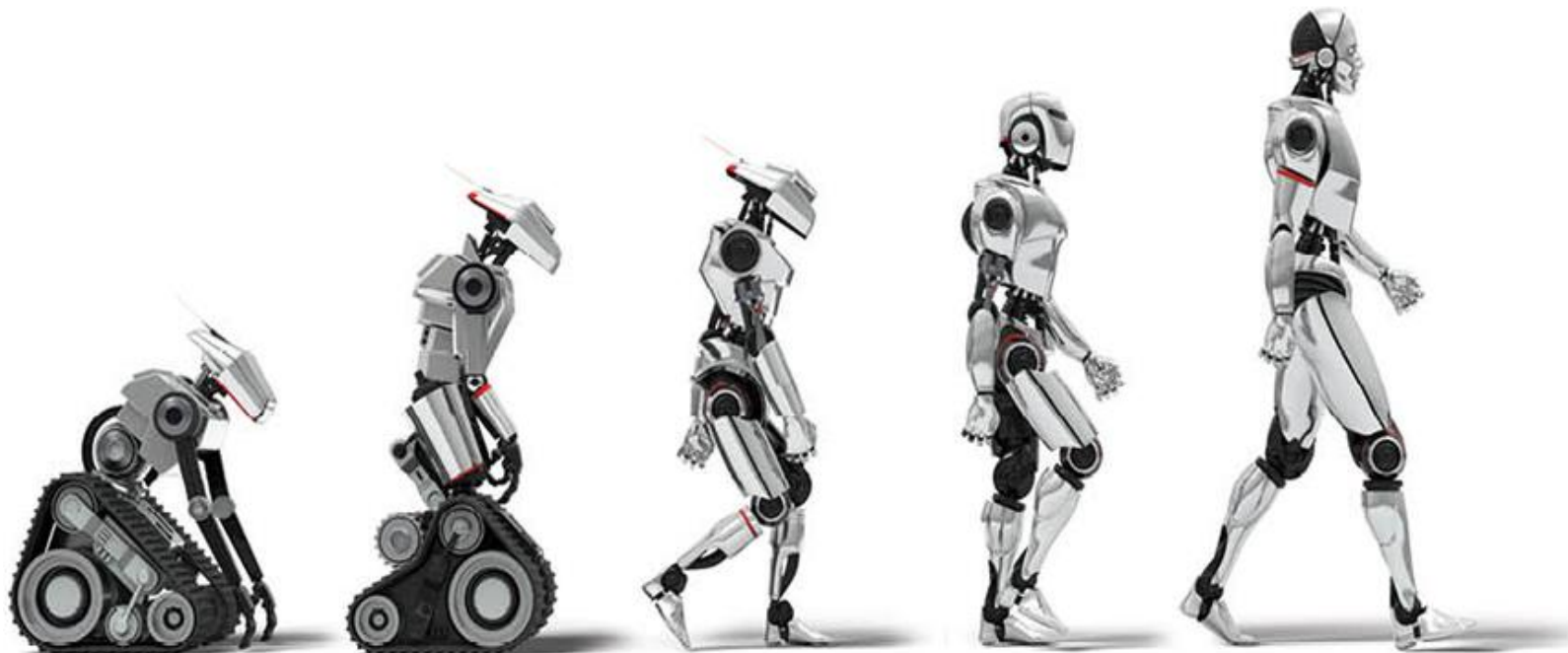


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## Exercises Robotics 10

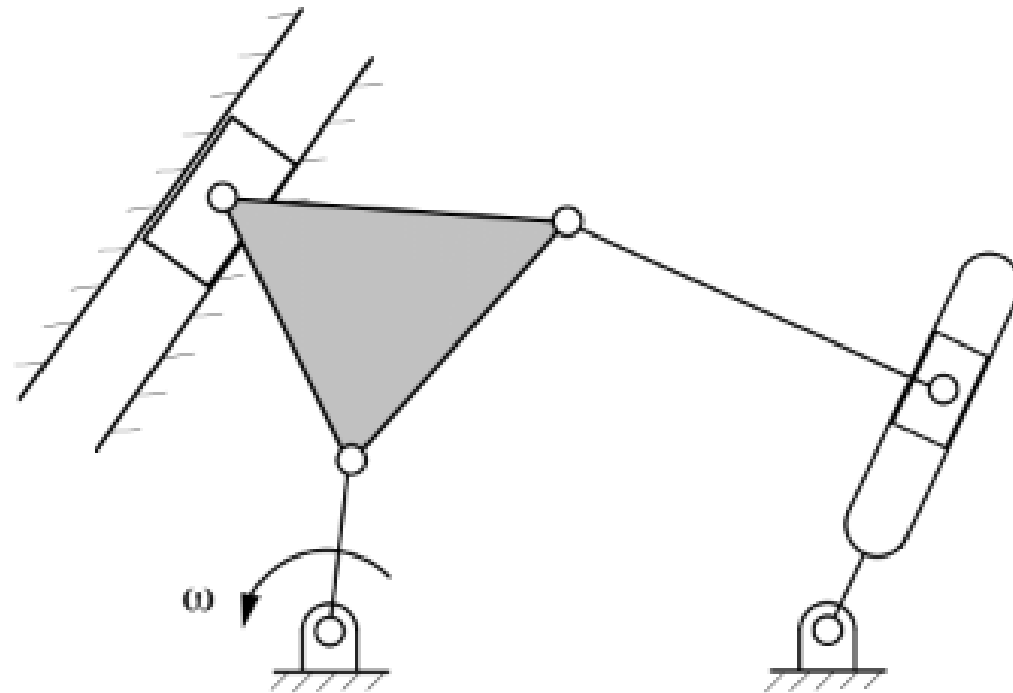


# Grübler's equation

Consider the following mechanism, define:

- The number of links;
- The number of kinematics joints stating the type and the constraints that they allow;

Apply then the Grübler's formula and show the number of DOF allowed by the mechanism.



# Rotation Matrix

A frame  $RF_B = \{\mathbf{O}_B, \mathbf{x}_B, \mathbf{y}_B, \mathbf{z}_B\}$  is displaced and rotated with respect to a fixed reference frame  $RF_A = \{\mathbf{O}_A, \mathbf{x}_A, \mathbf{y}_A, \mathbf{z}_A\}$ . The displacement is represented by the vector

$${}^A\mathbf{p}_{O_A O_B} = (3 \quad 7 \quad -1)^T \quad [\text{m}],$$

while the orientation of  $RF_B$  with respect to  $RF_A$  is represented by the following sequence of three Euler  $ZY'X''$  angles

$$\alpha = \frac{\pi}{4}, \quad \beta = -\frac{\pi}{2}, \quad \gamma = 0 \quad [\text{rad}].$$

For a given point  $P$ , provide the value of vector  ${}^A\mathbf{p}_{O_A P}$  knowing that its position with respect to frame  $RF_B$  is given by

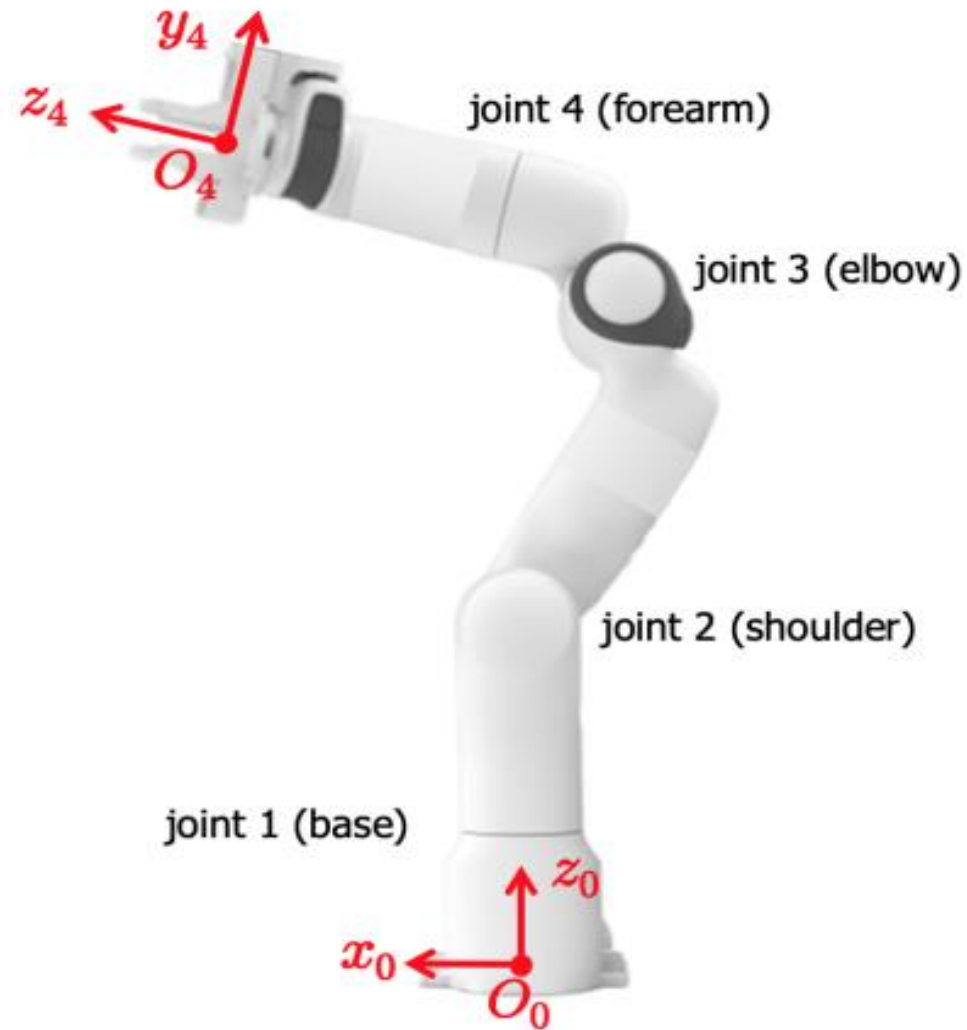
$${}^B\mathbf{p}_{O_B P} = (1 \quad 1 \quad 0)^T \quad [\text{m}].$$

# Rotation Matrix

$$\mathbf{R}_z(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{R}_y(\beta) = \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix}, \quad \mathbf{R}_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{pmatrix},$$

$${}^A\mathbf{T}_B = \begin{pmatrix} {}^A\mathbf{R}_B & A\mathbf{p}_{O_A O_B} \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} 0 & -\sqrt{2}/2 & -\sqrt{2}/2 & 3 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & 7 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

# Exercises D-H



# Exercises D-H

- a) Assign the link frames according to the Denavit-Hartenberg (DH) convention and complete the associated table of parameters, specifying the signs of all constant symbolic parameters.
- b) Write explicitly the resulting DH homogeneous transformation matrices  $A_1^0(q_1)$  to  $A_4^3(q_4)$  and compute in an efficient way the direct kinematics  $p_4 = p_4(q) \in \mathbb{R}^3$  for the position of the origin  $O_4$ .
- c) Sketch the robot in the stretched upward configuration and specify which is the associated configuration  $q_s$  in your DH convention. Compute then  $p_s = p_4(q_s)$ .
- d) In the configuration  $q_0 = 0$  determine the expression in the base frame of the absolute position of a Tool Center Point (TCP) which is defined in the end-effector frame by  $p_{4,TCP} = [0 \ 0.1 \ 0.2]^T [m]$ .

Thank you for your Attention!!!

