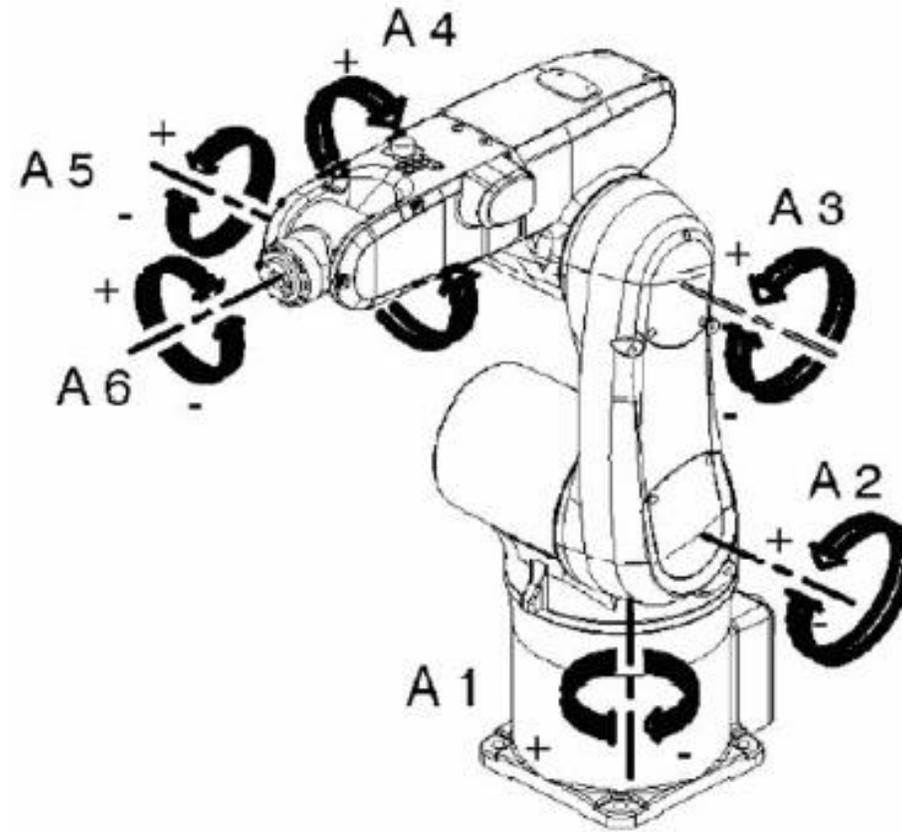




# Introduction to robotic components





# Hystorical notes

**Mechanics** is one of the most ancient disciplines, mainly based on practical applications in life.



**Moritz Geiger** (German Phylosopher 1880-1937)

«The man was the first in generating the circular motion in nature»



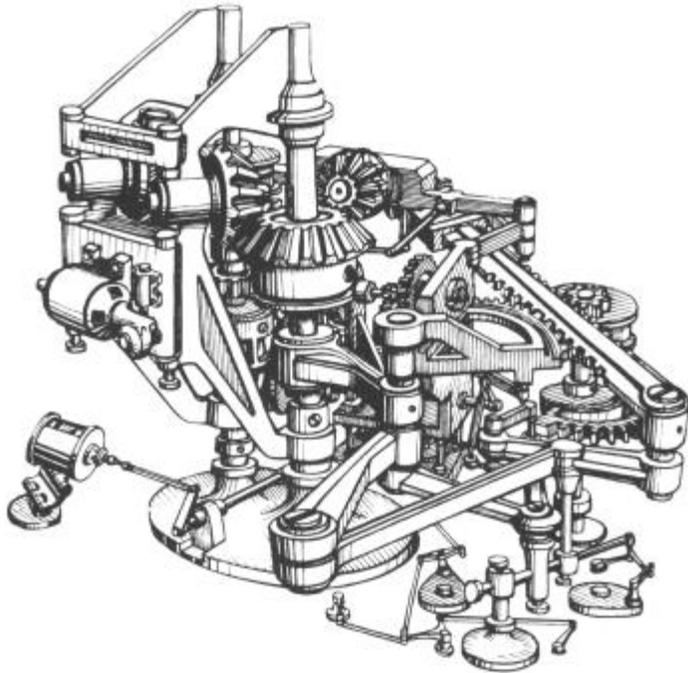
**Franz Reuleaux**(Prof. Berlin Technical Royal Academy 1829 –1905)

«mechanisms were conceived more for kinematics requests than energetic»



# Mechanism (definition)

- an assembly of moving parts performing a complete functional motion.

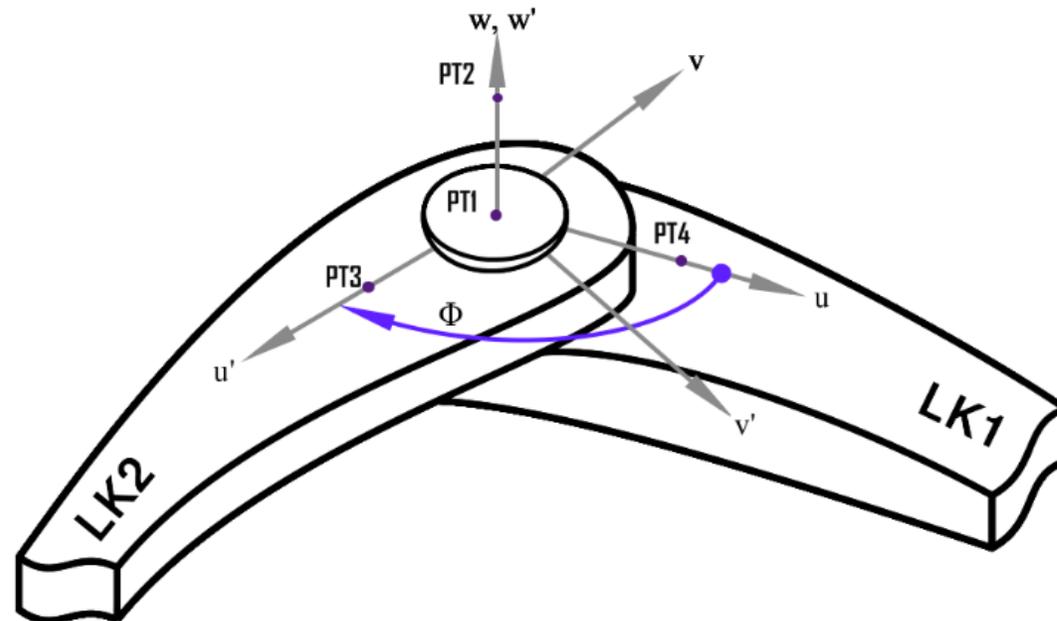


the **degree of freedom (DOF)** the number of independent parameters that defines a mechanism's configuration.

The number of DOFs depends on the number of composing links and connecting joints of the mechanism.

# Components and structures of a mechanism

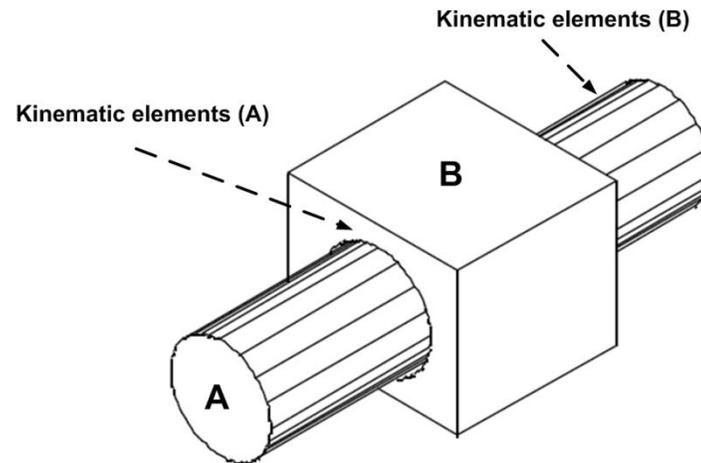
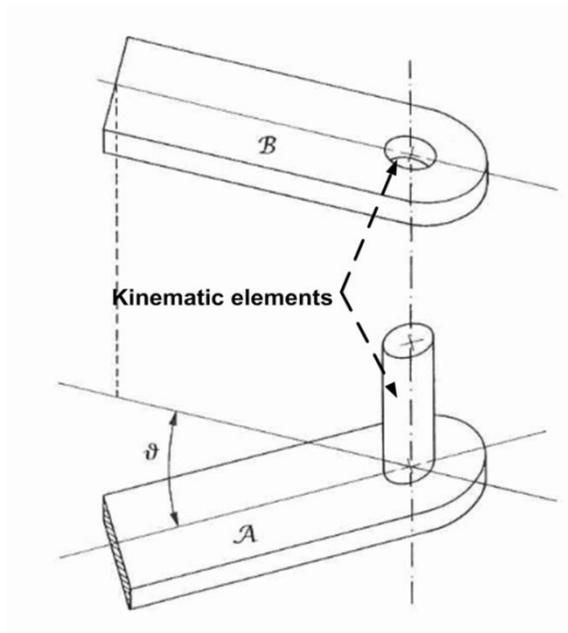
- The individual bodies making up a mechanism are called **links** or **members**
- Two or more members/links connected together by a joint and allowed to relatively move are a **kinematic chain**
- Relative motion is permitted by the **constraints** or **joints**



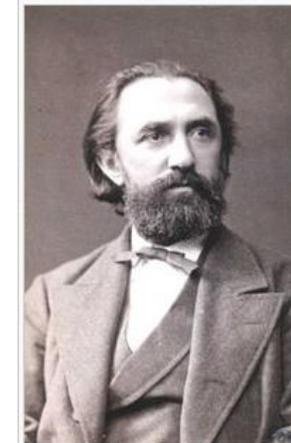


# Classification of kinematic pairs

- The contact element of a single link is called **kinematic element**
- two members assembled by their kinematic elements limiting their relative motion are a **kinematic pair**

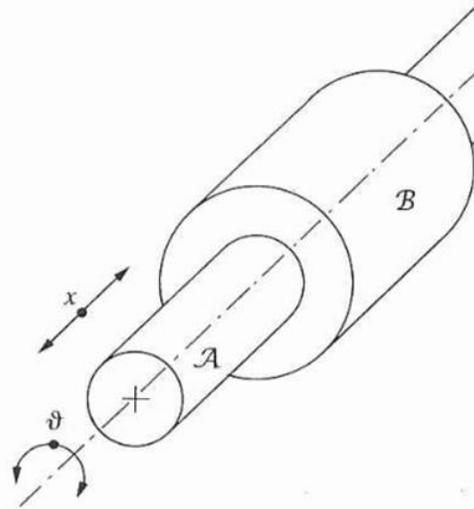


- **kinematic pair** can be classified according Reuleaux

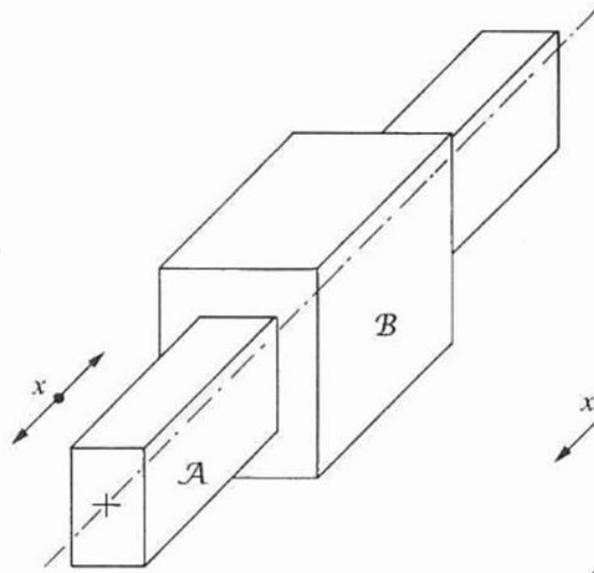


# Reuleaux Classification of kinematic pairs

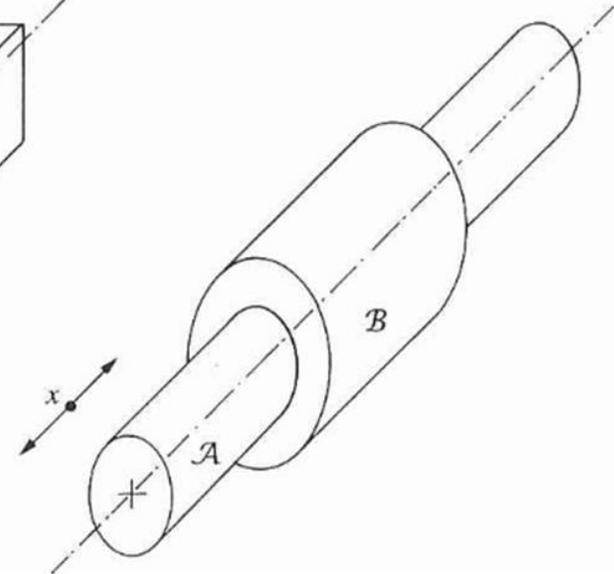
- **LOWER ORDER (LO) KINEMATIC PAIR:**  
-Characterized by a **SURFACE** contact



**cilindircal**



**prismatic**

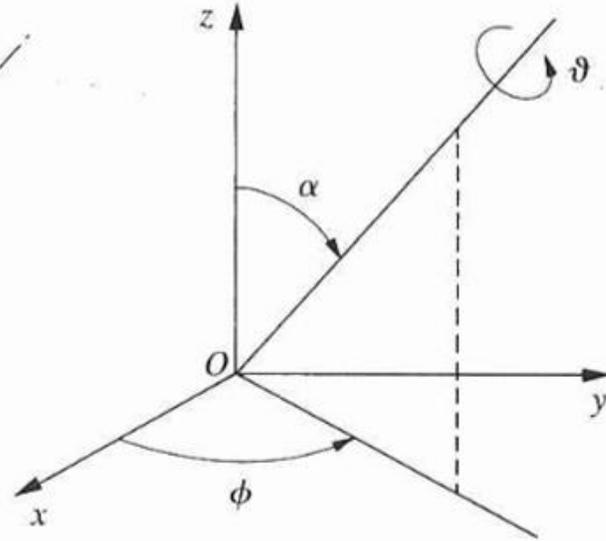
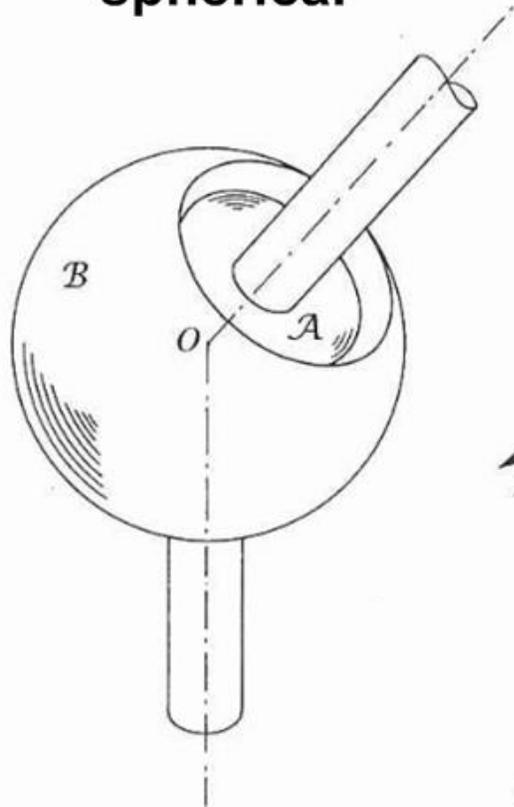


**elliptical**

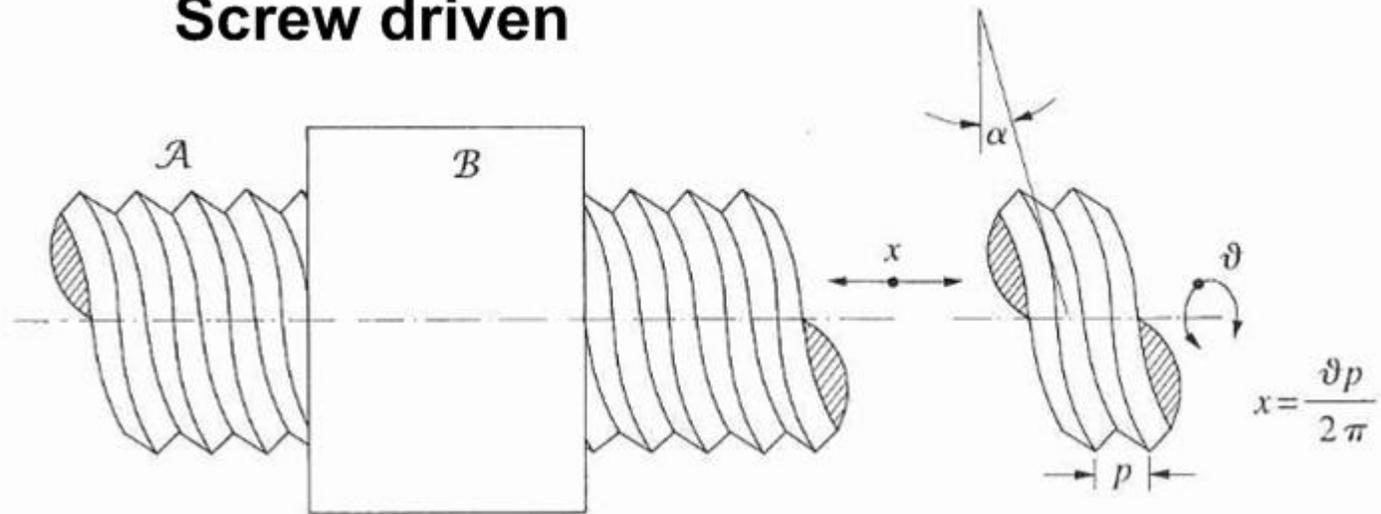


# Classification of kinematic pairs (Low order)

**spherical**



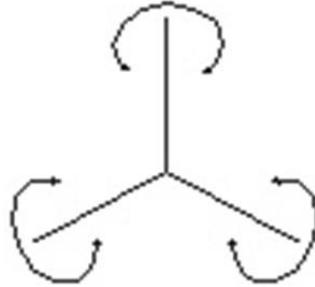
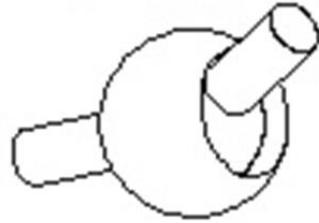
**Screw driven**



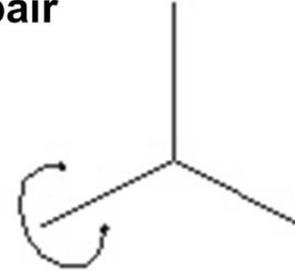


# Low order kinematic pairs

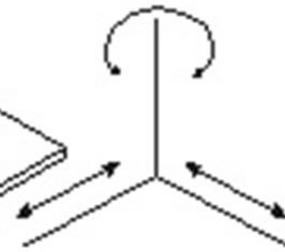
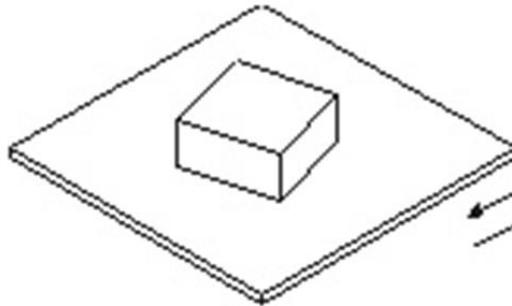
**Spherical pair**



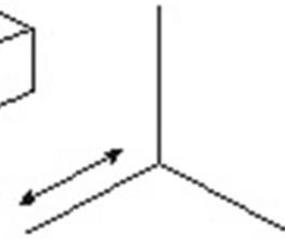
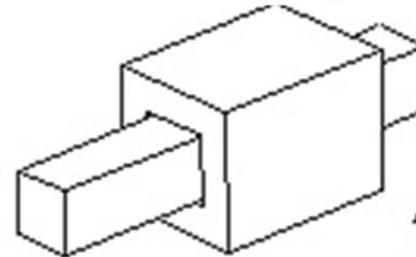
**Revolute pair**



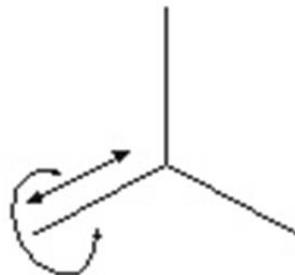
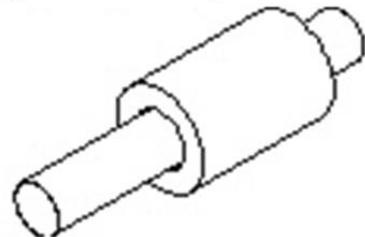
**Planar pair**



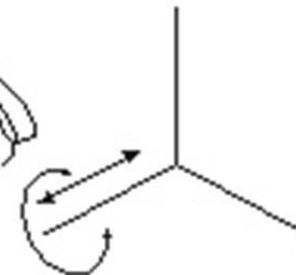
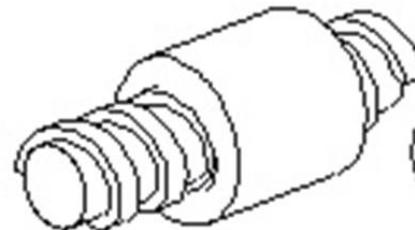
**Prismatic pair**



**Cylindrical pair**

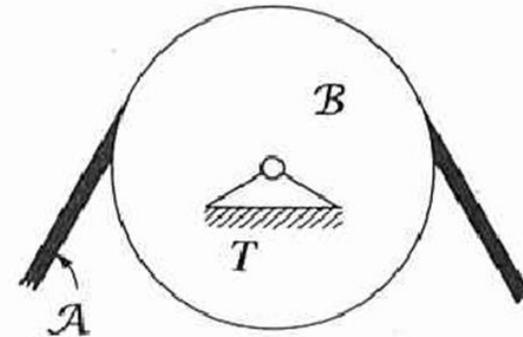
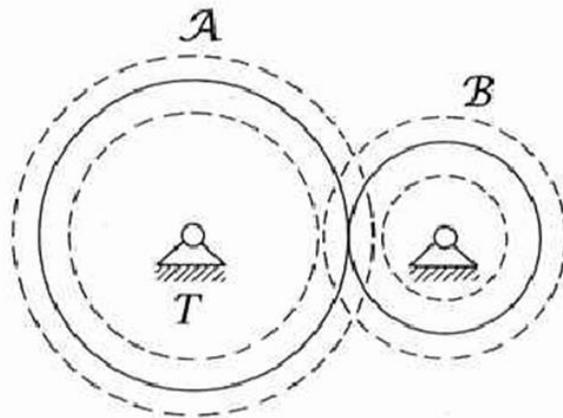
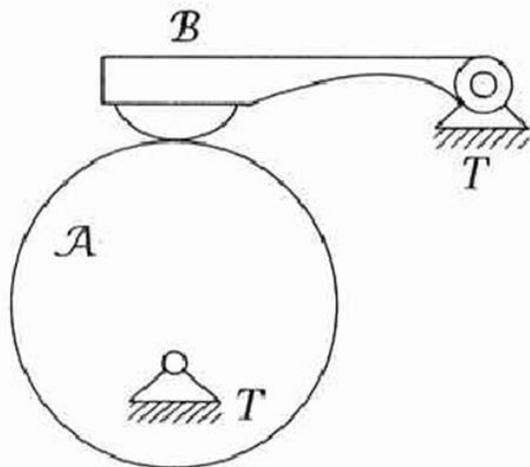
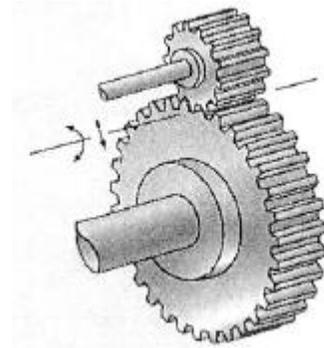
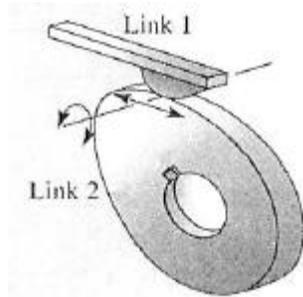


**Screwpair**



# Classification of kinematic pairs

- **HIGHER ORDER (HO) KINEMATIC PAIR:**  
-Characterized by a **LINE** or **POINT** of contact

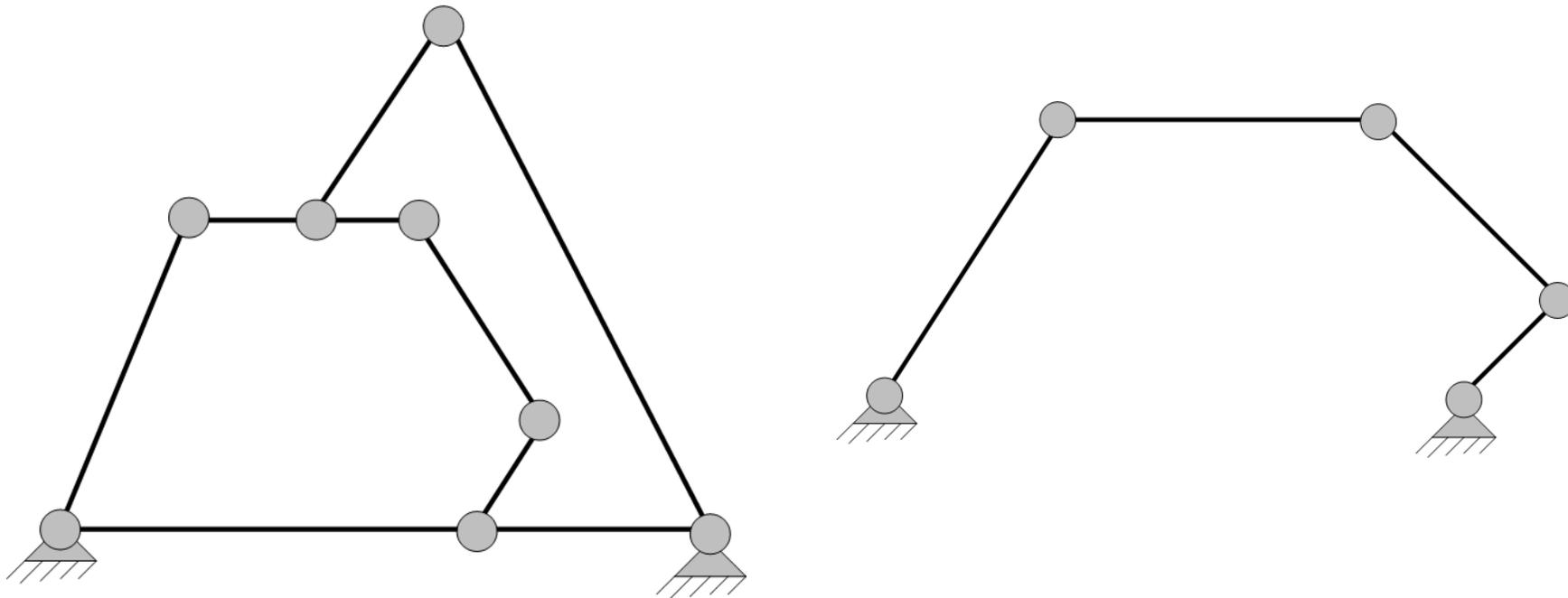




# Closed/open loop kinematic chain

**A kinematic chain (KC) is an assembly of links connected by joints**

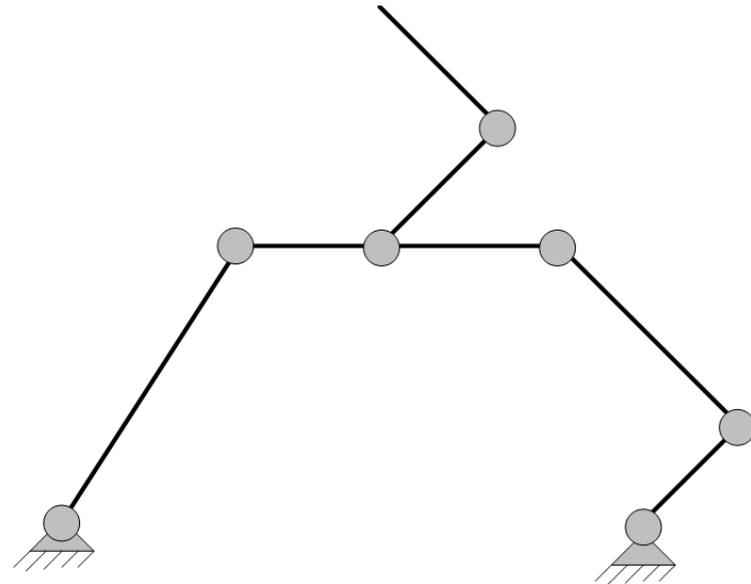
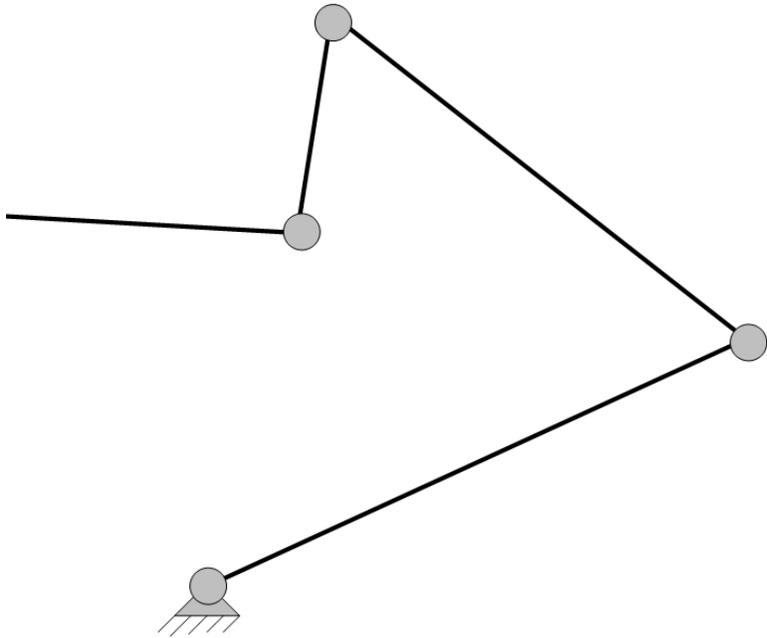
-when every link in a KC is connected to every other link by atleast two distinct paths forming one or more closed loop then the KC is called **closed-loop KC**





# Open loop kinematic chain

-if every link is connected to every other link by one and only one path then one has an **'open loop'** KC



-it is also possible having hybrid KC (close and open loop connected)



# Degrees of freedom of a planar 2D mechanism

In the kinematic synthesis of a mechanism the number of degrees of freedom is the first concern.

The **degree of freedom (DOF)** of a mechanical system is the number of independent parameters that defines its configuration.

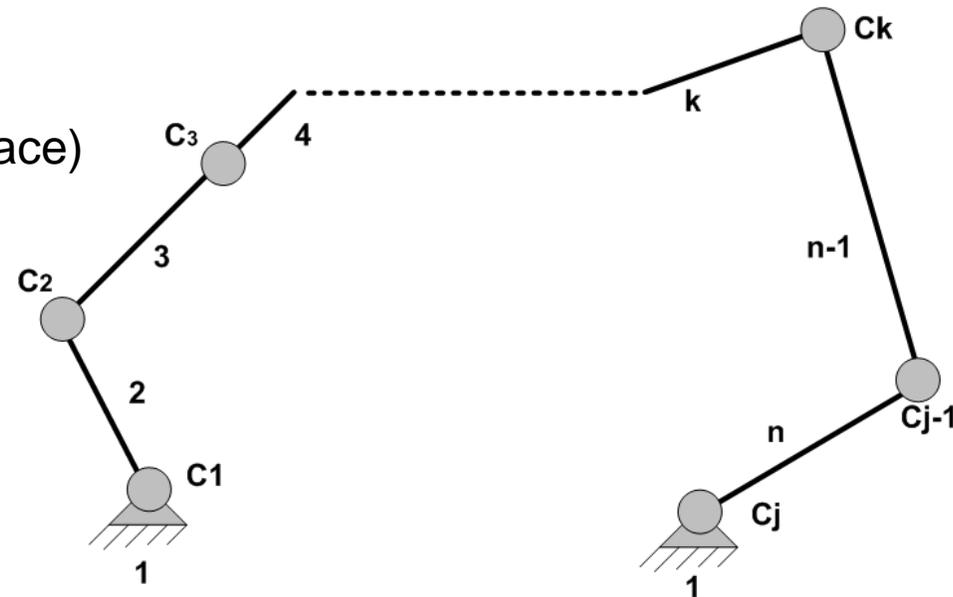
$$F = \lambda(n - 1) - \sum_{i=1}^j c_i \quad \text{Grübler Formula}$$

$\lambda$  DOFs of the operating space of the mechanism (3 if on the plane...6 in space)

$n$  Number of links

$j$  Number of kinematic pairs

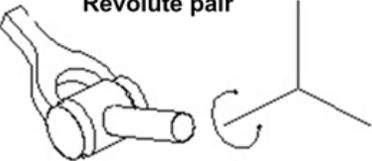
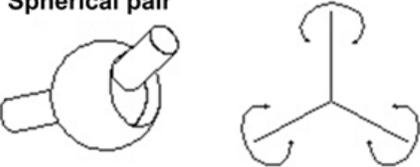
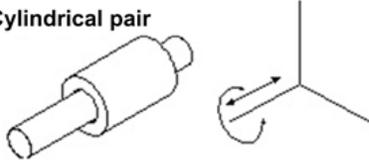
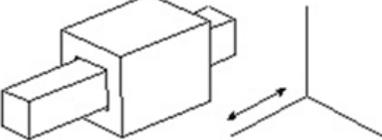
$c_i$  Degree of constraint of the  $i$ -th kinematic pair





# Degree of constraint ( $C_i$ )

Intuitively the DOFs of a mechanism is equal to the degrees of freedom associated with all the moving links minus the number of constraints imposed by the kinematic pairs.

Kinematic pair	DOC planar workspace	DOC spatial workspace
<p>Revolute pair</p> 	$C_i = 2$	$C_i = 5$
<p>Spherical pair</p> 	$C_i = 2$	$C_i = 3$
<p>Cylindrical pair</p> 	$C_i = 2$	$C_i = 4$
<p>Prismatic pair</p> 	$C_i = 2$	$C_i = 5$



# Example of planar mechanism

$$F = 3(n - 1) - 2J_L - J_H$$

$J_H =$  Number of higher order kinematic pairs

$J_L =$  Number of lower order kinematic pairs

$n =$  Number of links

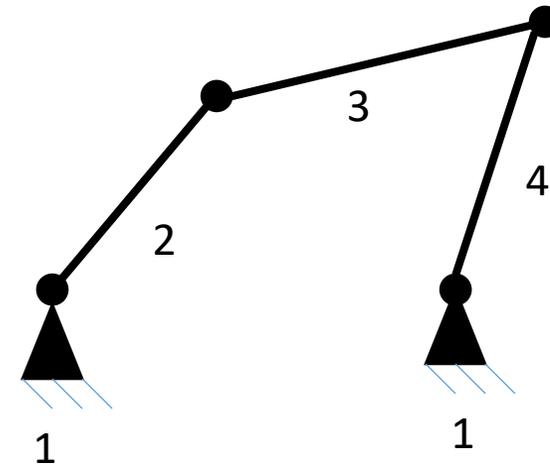
$\lambda = 3$  Number of DOFs in the planar workspace

$$F = 3(n - 1) - 2J_L - J_H = 1$$

$$J_H = 0$$

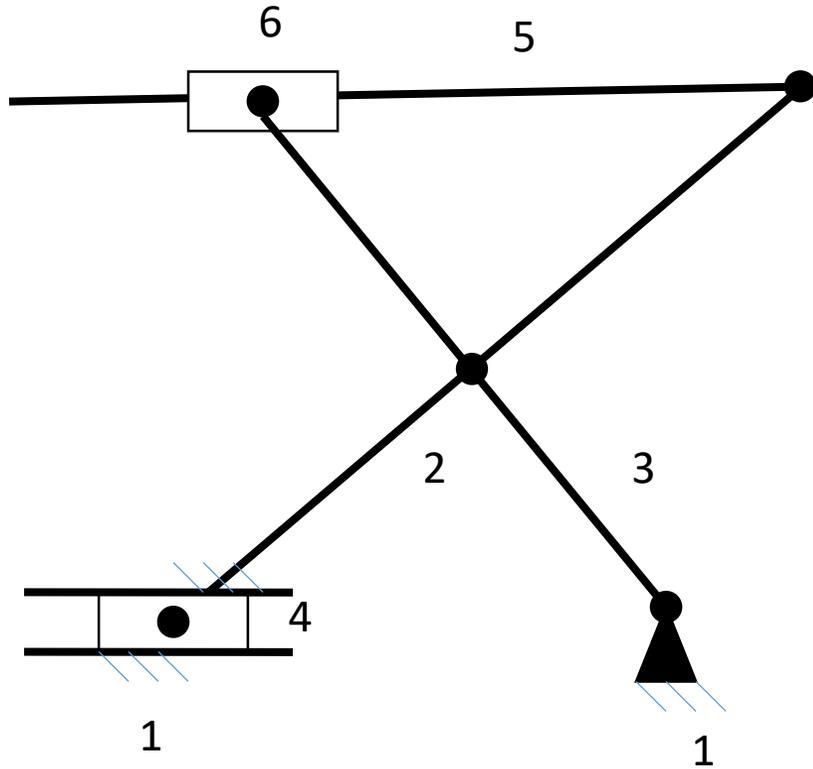
$$J_L = 4$$

$$n = 4$$





# Example of planar mechanism

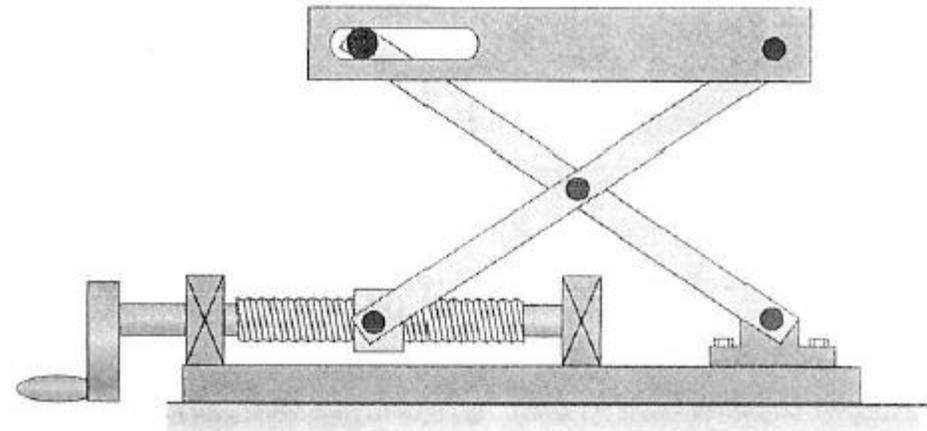


$$F = 3(n - 1) - 2J_L - J_H = 1$$

$$J_H = 0$$

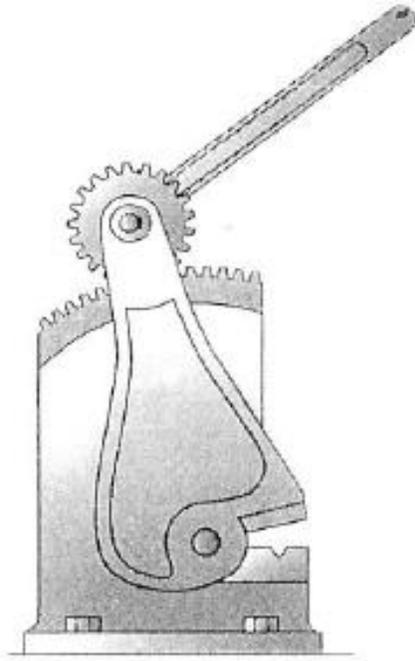
$$J_L = 7$$

$$n = 6$$





# Planar mechanism higher order kinematic pair

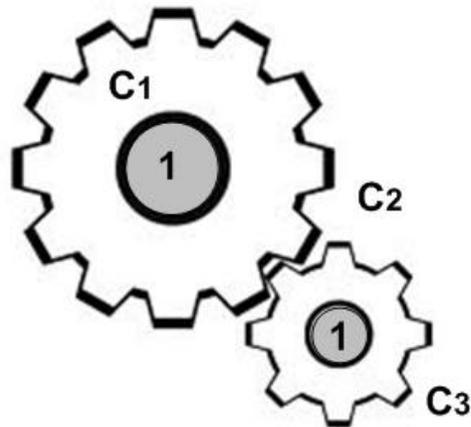
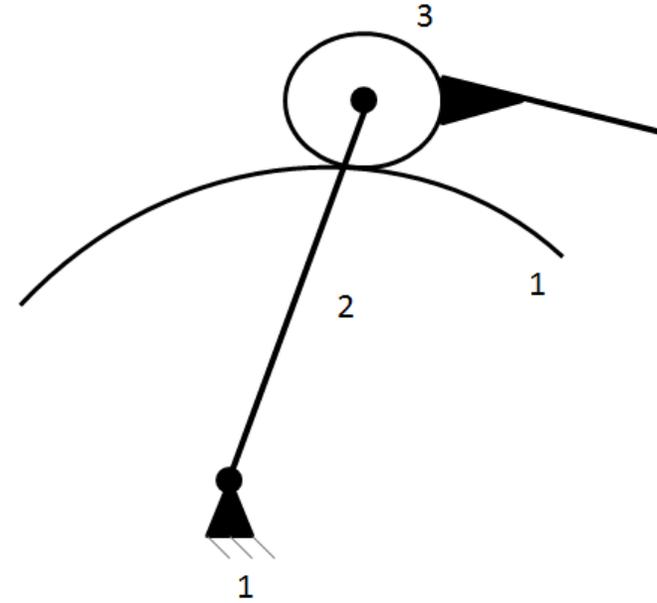


$$F = 3(n - 1) - 2J_L - J_H = 1$$

$$J_H = 1$$

$$J_L = 2$$

$$n = 3$$

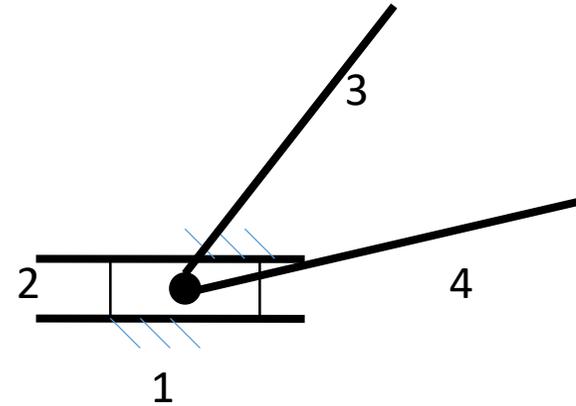
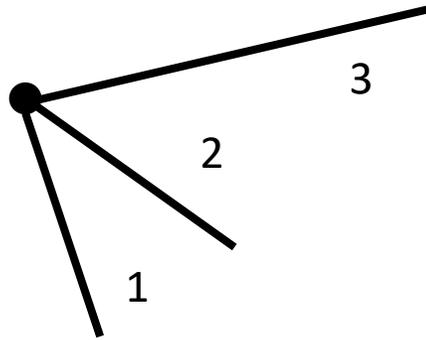


**Gears are higher order kinematic pair, interacting by a contact point on the two active teeth.**



# Coincident joints

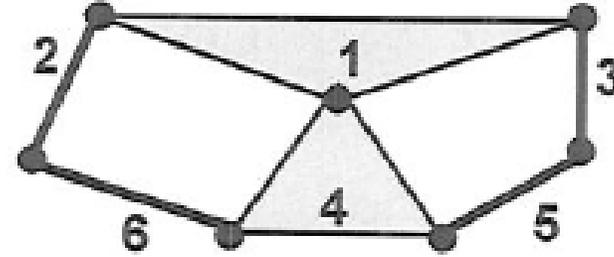
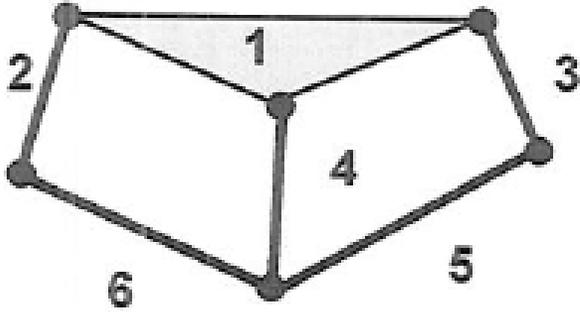
- Some mechanism have three kinematic pairs concurring on the same joint



**Such configuration can lead to confusion for mechanical modeling**



# Coincident joints



$$F = 3(n - 1) - 2J_L - J_H = 1$$

$$J_H = 0$$

$$J_L = 7$$

$$n = 6$$

In this condition the mechanism can be decomposed and applying the Grübler Formula would allow to find the exact number of degrees of freedom



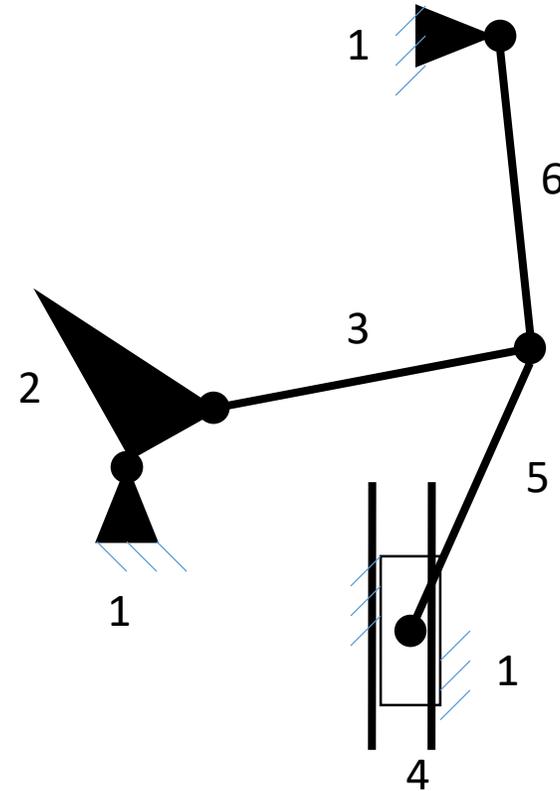
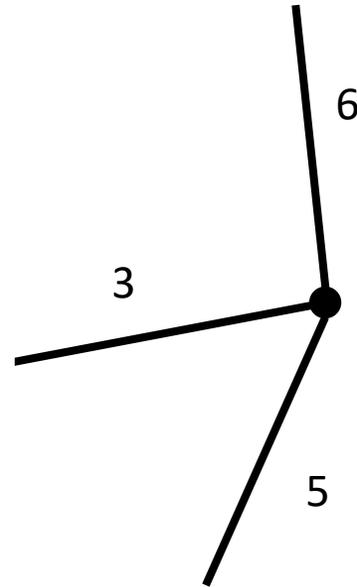
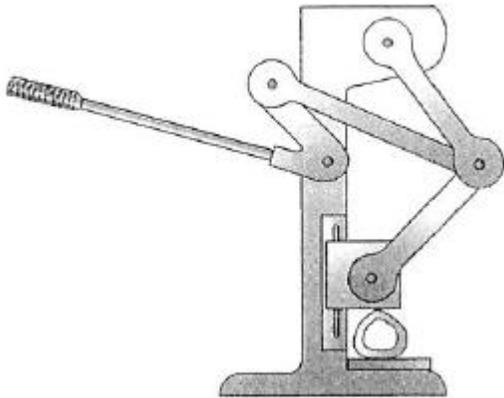
# Coincident joints

$$F = 3(n-1) - 2J_L - J_H = 1$$

$$J_H = 0$$

$$J_L = 7$$

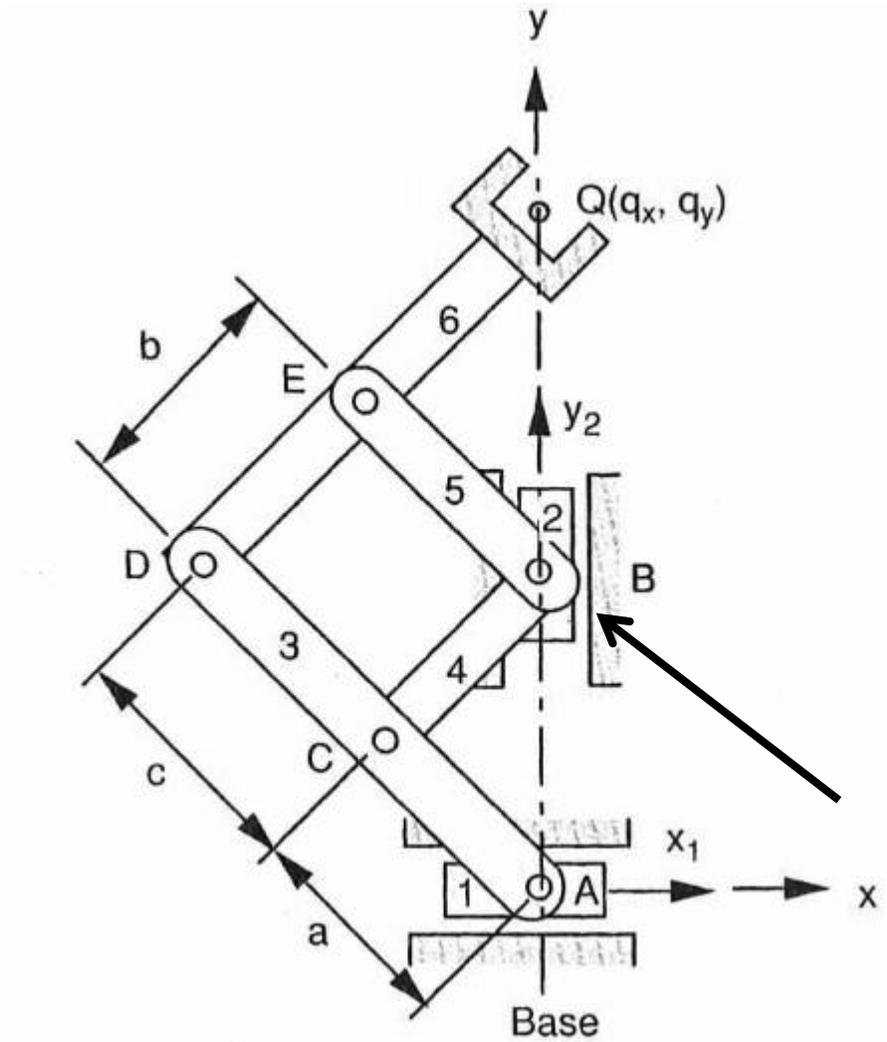
$$n = 6$$



A joint with three concurring links can be decomposed as two revolute kinematic pairs



# Coincident joints



$$F = 3(n - 1) - 2J_L - J_H = 2$$

$$J_H = 0$$

$$J_L = 8$$

$$n = 7$$

The coincident joint can be assumed as 3 revolute kinematic pairs



# Example of spatial mechanism

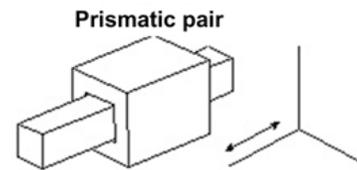
For spatial mechanism too it is possible to evaluate by Grübler Formula the number of Degrees of Freedom.

$$F = \lambda(n - 1) - 5C_1 - 4C_2 - 3C_3 - 2C_4$$

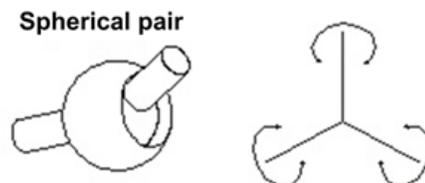
$n$  = Number of links

$\lambda$  = 6 Number of DOFs in the spatial workspace (3 translations and 3 rotations)

Where  $C_1$  is the number of kinematic pairs suppressing 5 DOFs.



Where  $C_3$  is the number of kinematic pairs suppressing 3 DOFs.





# Example of spatial mechanism

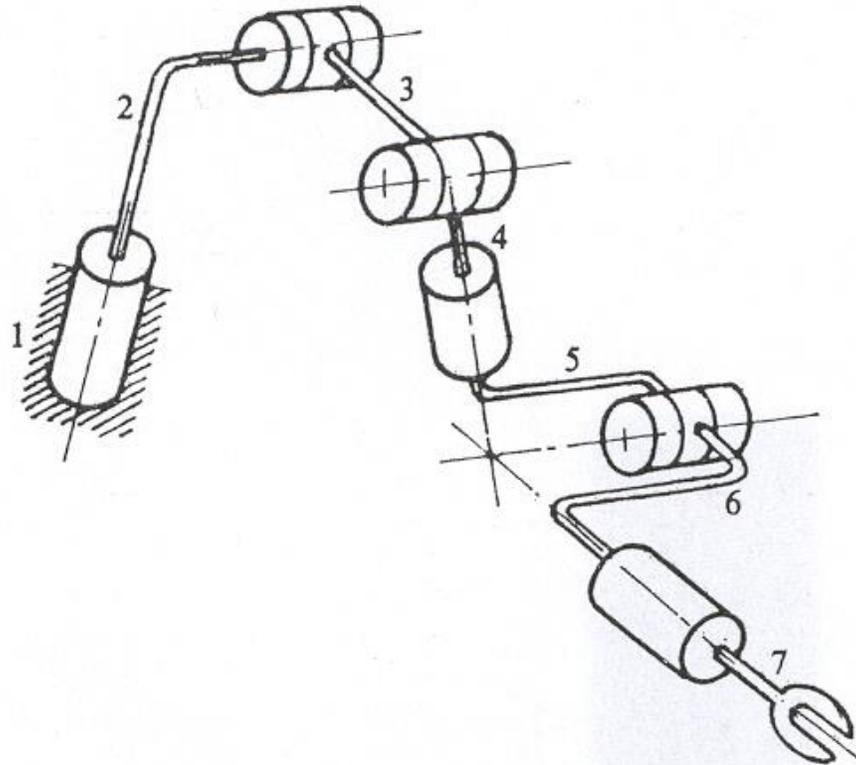
This is a spatial mechanism able to move according 3 different axes:

$$F = \lambda(n-1) - 5C_1 - 4C_2 - 3C_3 - 2C_4 = 36 - 30 = 6$$

$$n = 7$$

$$\lambda = 6$$

$$C_1 = 6$$





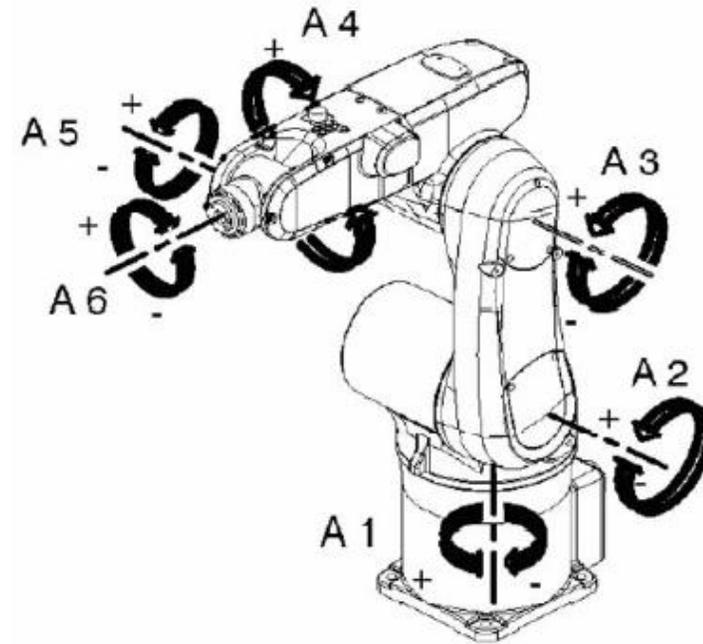
# Robotics uses Machine theory

A robotic system is composed by multiple rigid bodies (**LINKS**) moving amongst each other by means of constraints which are called **JOINTS**.

- Rigid segments can move with respect to each other due to joints



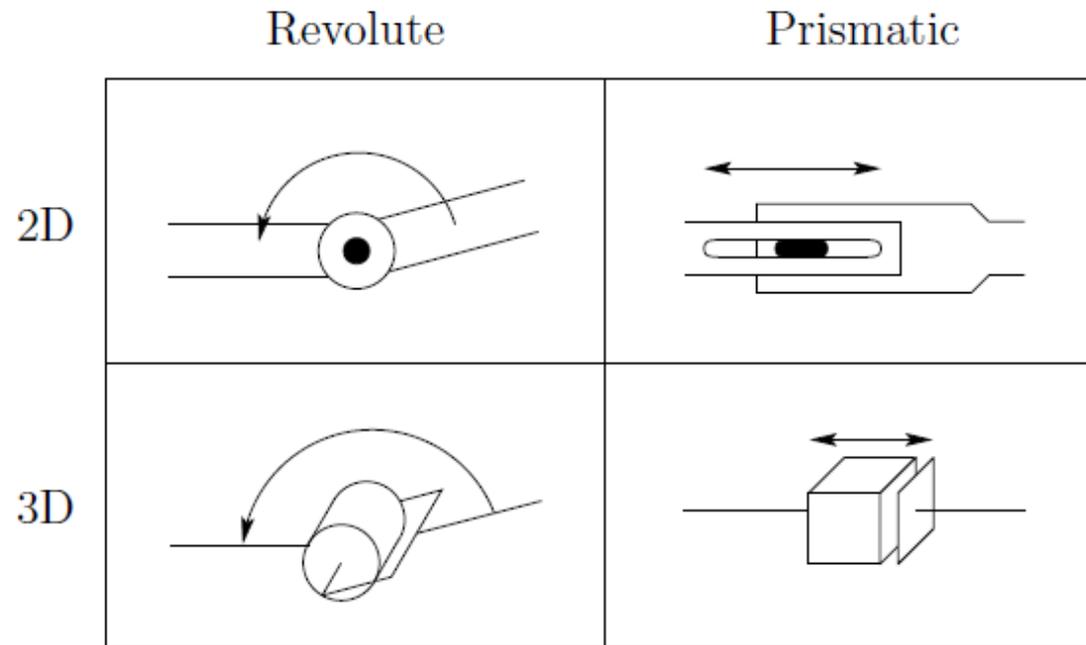
KUKA sixx R850





# Symbolic representation of robots

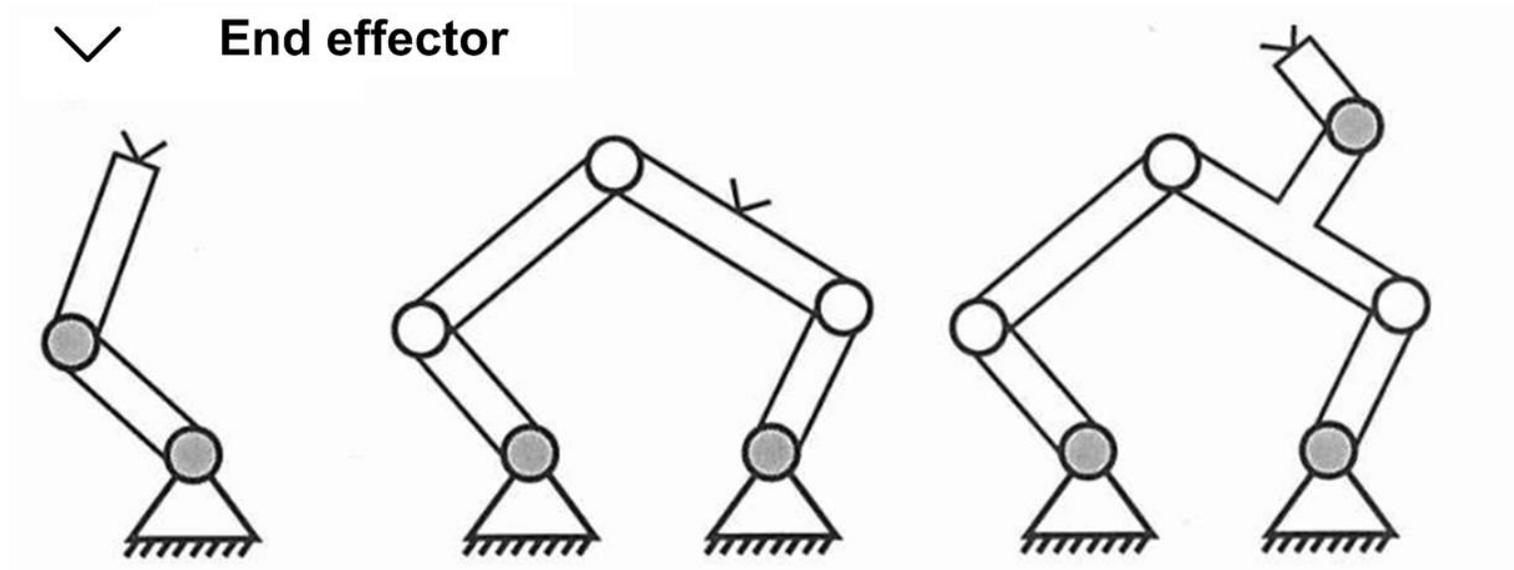
- Robot Manipulators are composed of links connected by joints into a kinematic chain.
- Joints are typically rotary (revolute) or linear (prismatic).
- A revolute joint is like a hinge and allows relative rotation between two links.
- A prismatic joint allows a linear relative motion between two links.
- We use the convention (R) for representing revolute joints and (P) for prismatic joints



# Robot classification

## Structural topology

- **SERIAL (open loop)**
- **PARALLEL (closed loop)**
- **HYBRID (open+closed loop)**



- Actuated joint
- UnActuated joint



# Serial robot

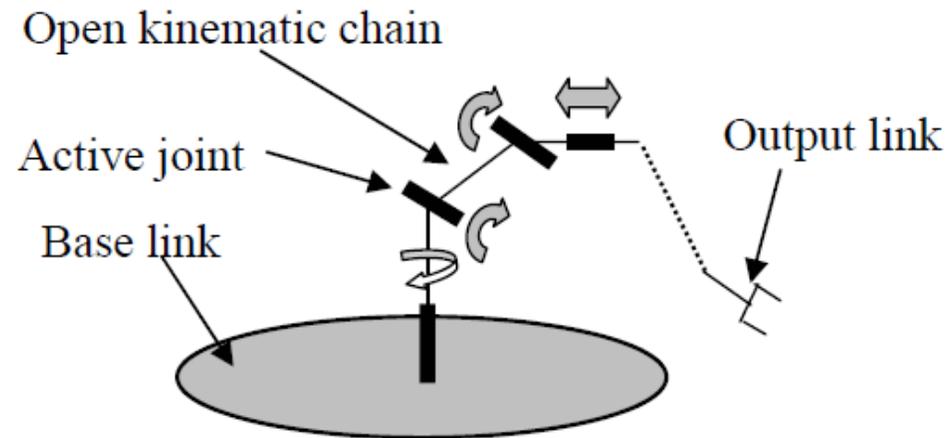
- anthropomorphic architecture
- open loop kinematic chain.

## Advantage:

-large workspace and high dexterity

## Disadvantage:

- low precision, poor force
- low payload-to-weight ratio,
- motors inertia
- large number of moving parts.

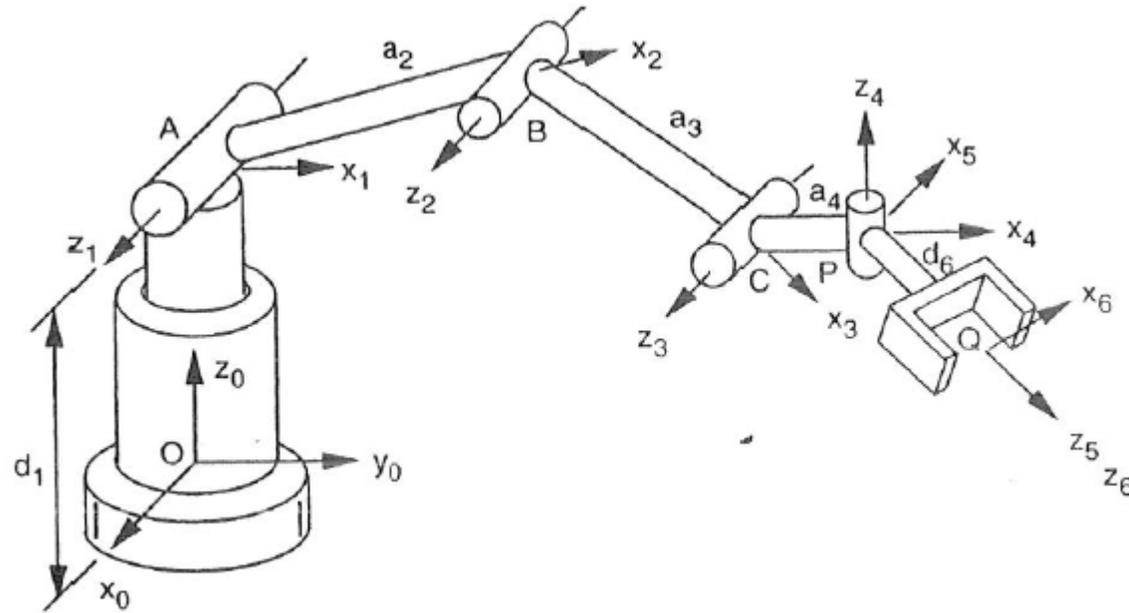




# Serial robots



# Open Kinematic chain (serial robot)



$$F = \lambda(n-1) - 5C_1 - 4C_2 - 3C_3 - 2C_4 = 30 - 25 = 5$$

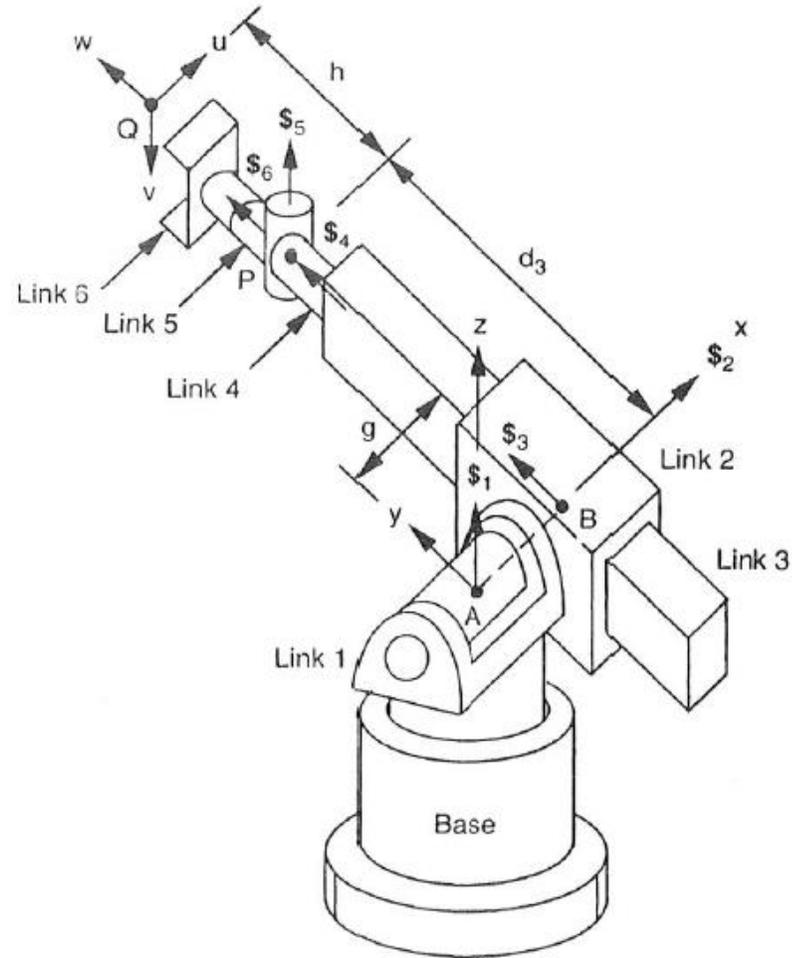
$$n = 6$$

$$\lambda = 6$$

$$C_1 = 5$$



# Open Kinematic chain (serial robot)



$$F = 6(n - 1) - 5J_L - J_H = 5$$

$$J_{Higher} = 0$$

$$C_1 = J_{Lower} = 5$$

$$n = 6$$

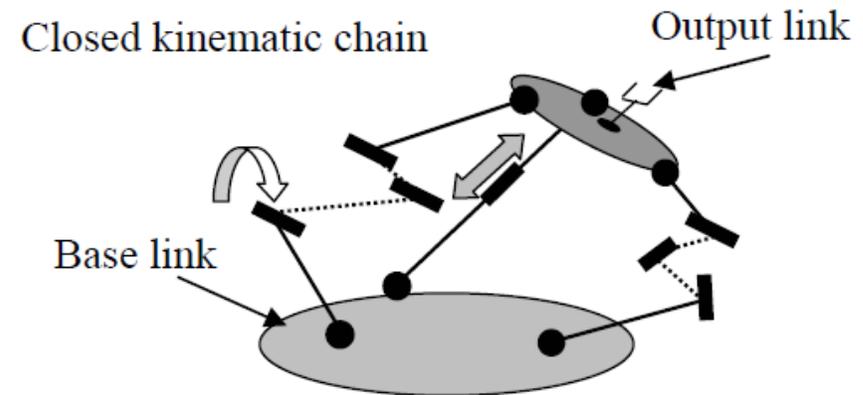
- non-anthropomorphic architecture
- closed kinematic chains

## Advantage:

- high accuracy,
- low inertia of moving parts
- high agility, and simple solution for the
- load is shared by several kinematic chains

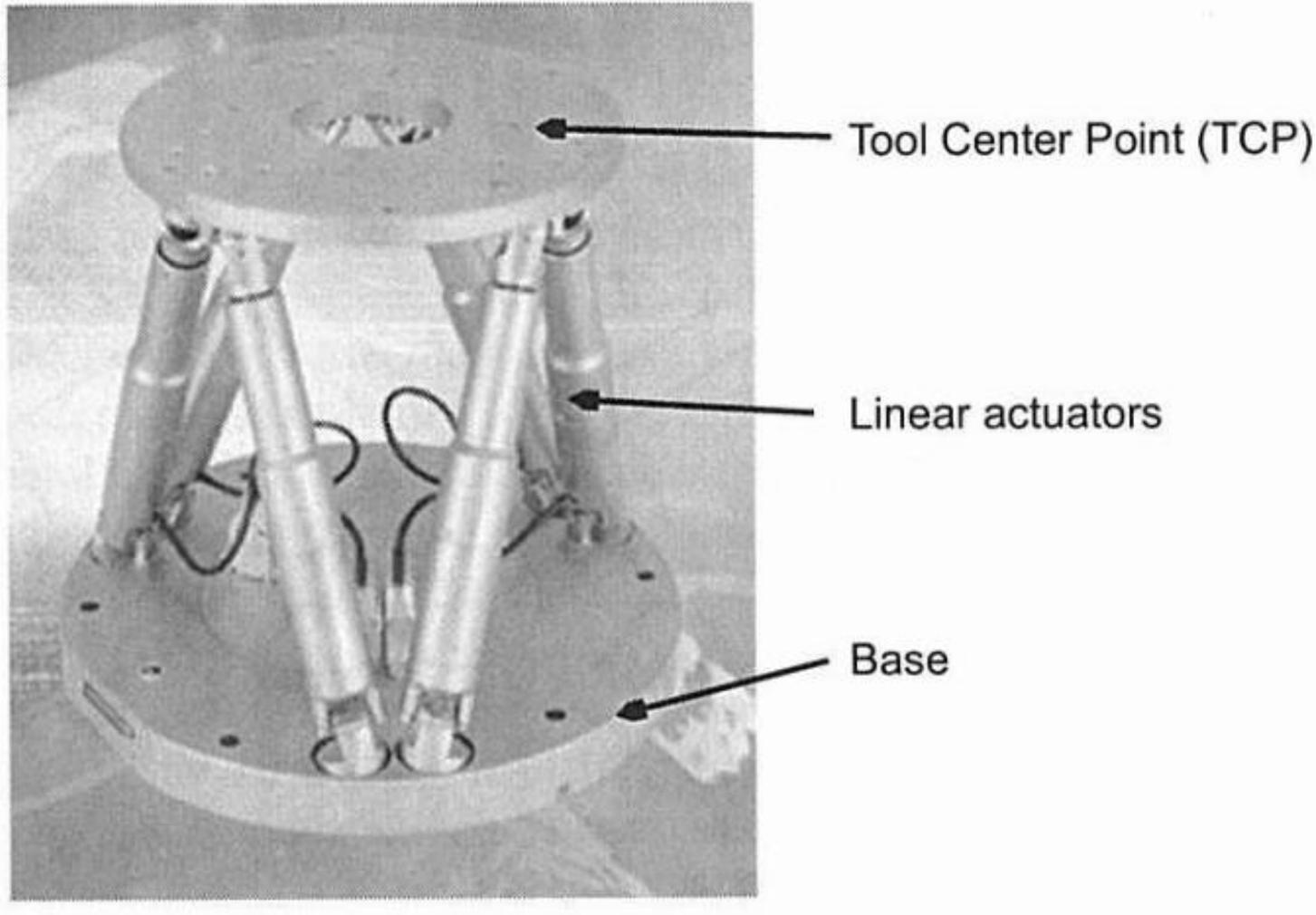
## Disadvantages:

- limited work volume,
- low dexterity
- complicated direct kinematics solution
- high rigidity and high payload-to-weight ratio





# PARALLEL ROBOT



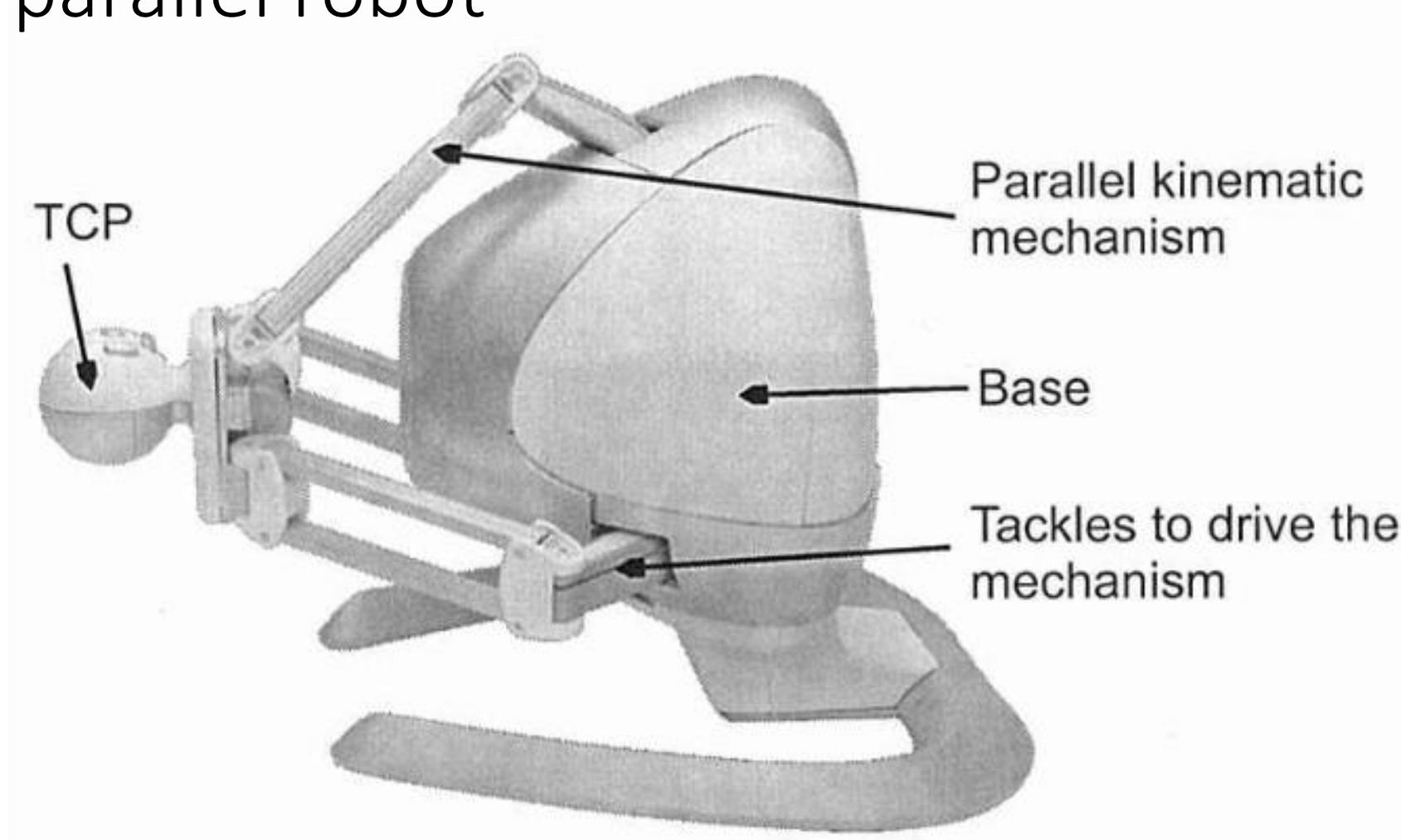


# PARALLEL ROBOT



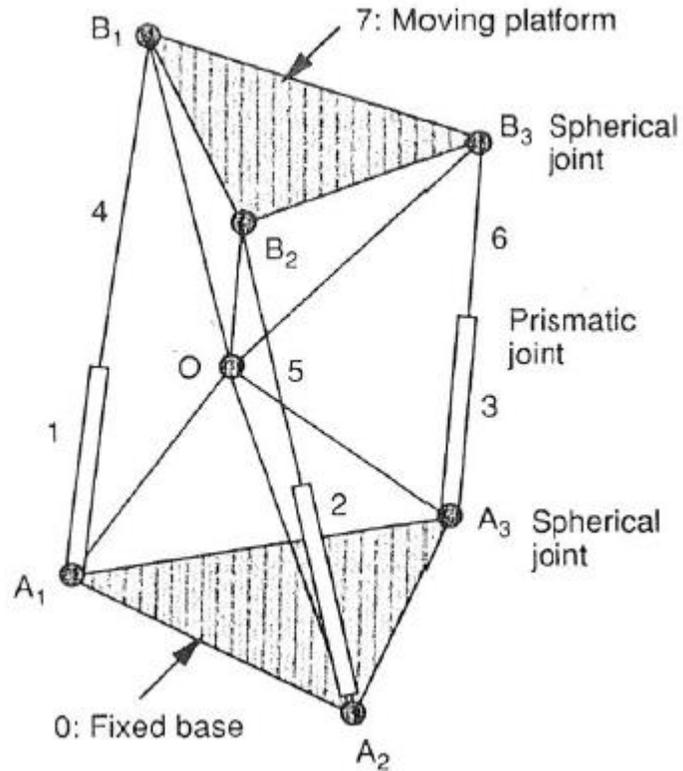


# Closed Kinematic chain parallel robot





# closed Kinematic chain (serial robot)



$$F = 6(n-1) - 5J_P - 3J_S - J_H = 6????$$

$$C_3 = J_S = 7$$

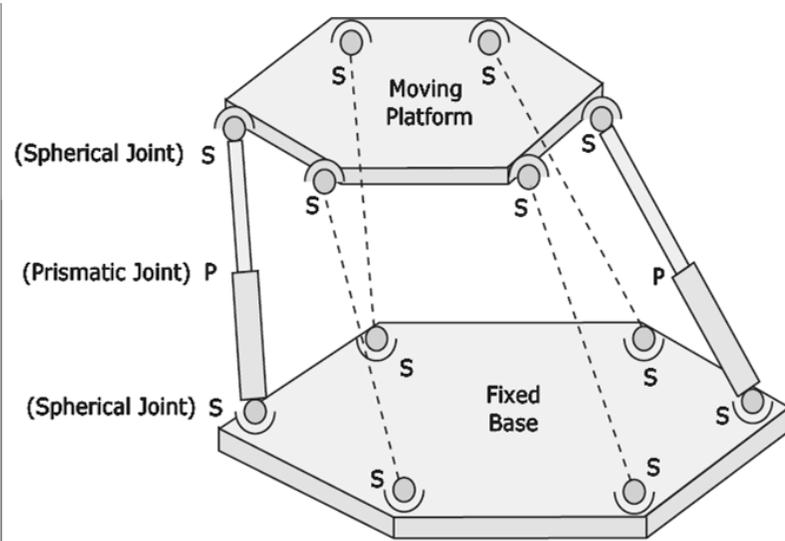
$$C_1 = J_P = 3$$

$$n = 8$$

The real actuated DOFs are only 3 because there are 3 underactuated DOFs of the «legs».



# closed Kinematic chain (serial robot)



$$F = 6(n - 1) - 5J_P - 3J_S - J_H = 12??$$

$$C_3 = J_S = 12$$

$$C_1 = J_P = 6$$

$$n = 12 + 2$$

The real actuated DOFs are only 6 because there are 6 underactuated DOFs of the «legs».

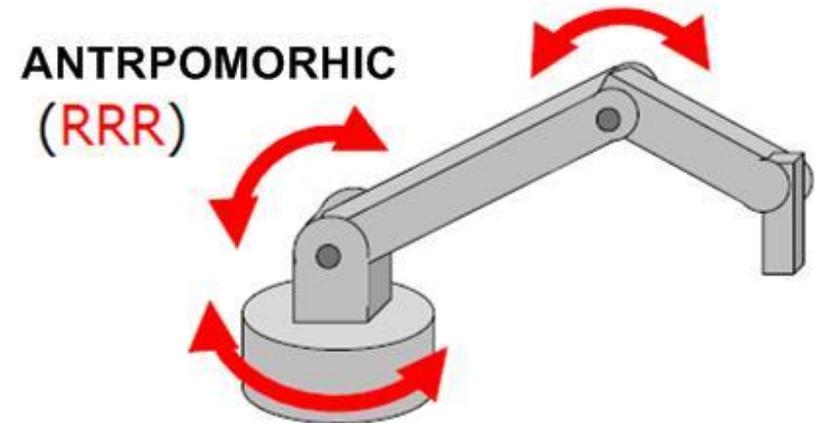
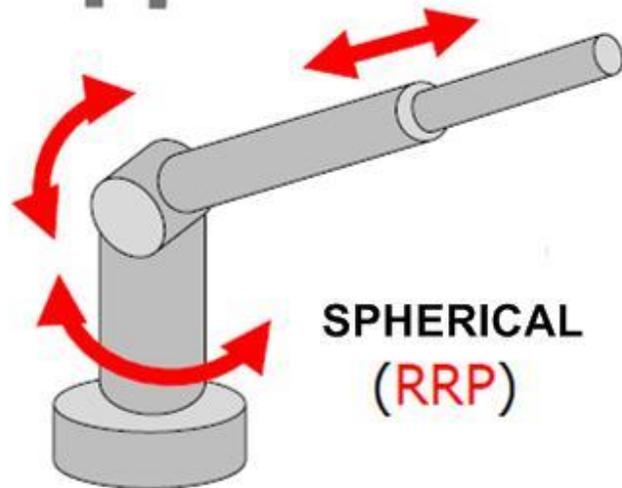
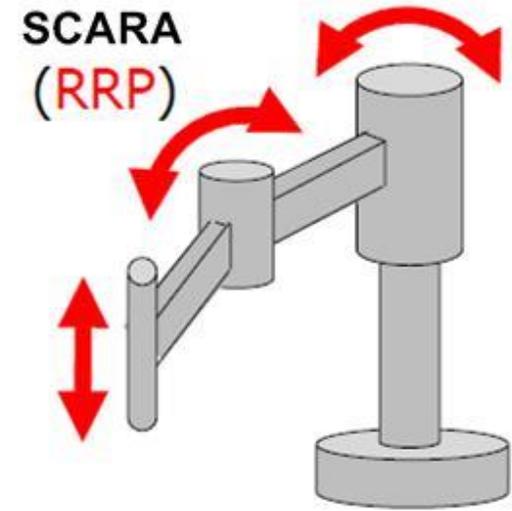
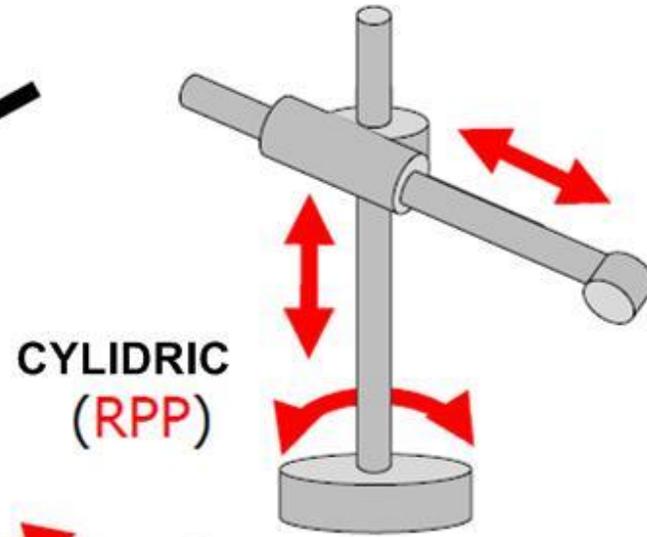
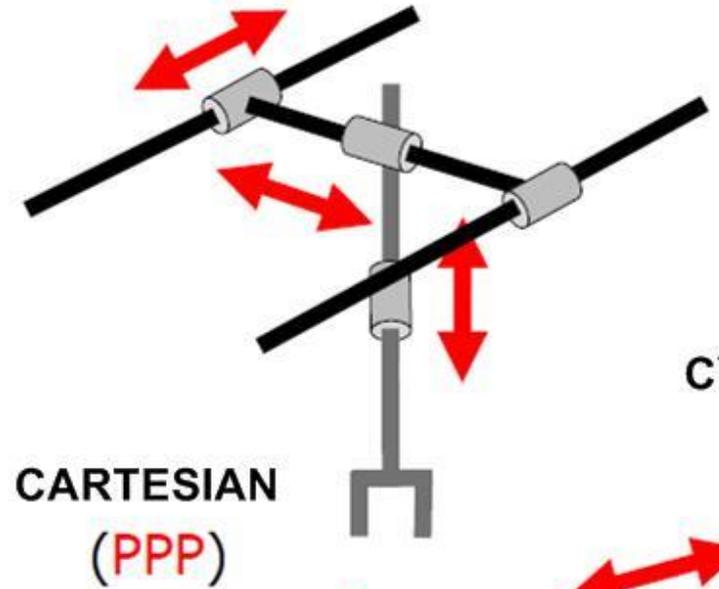


# Serial VS parallel



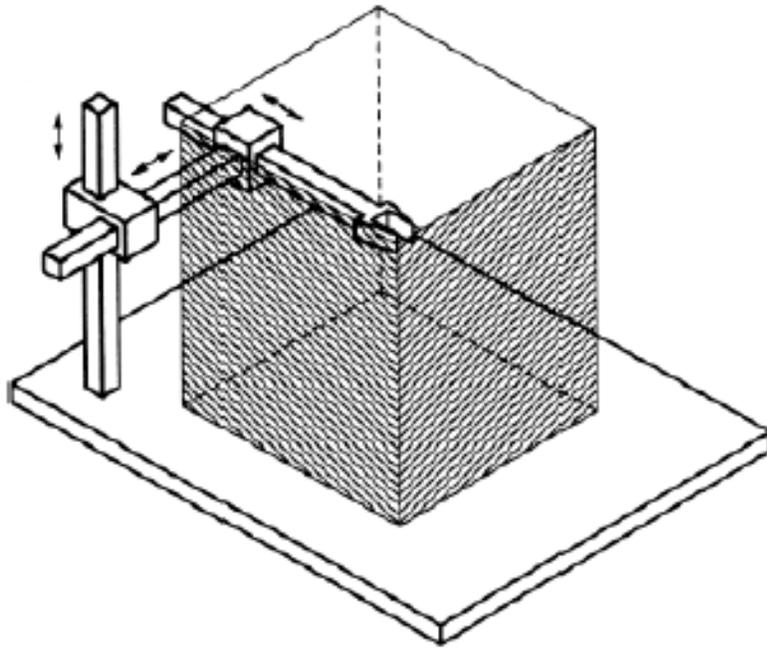


# Structure denomination

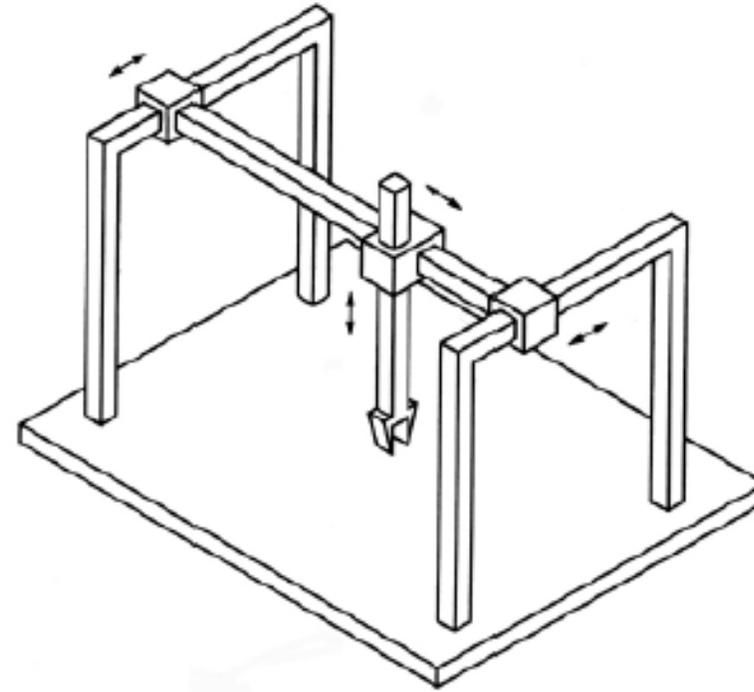




# Robot classification workspace



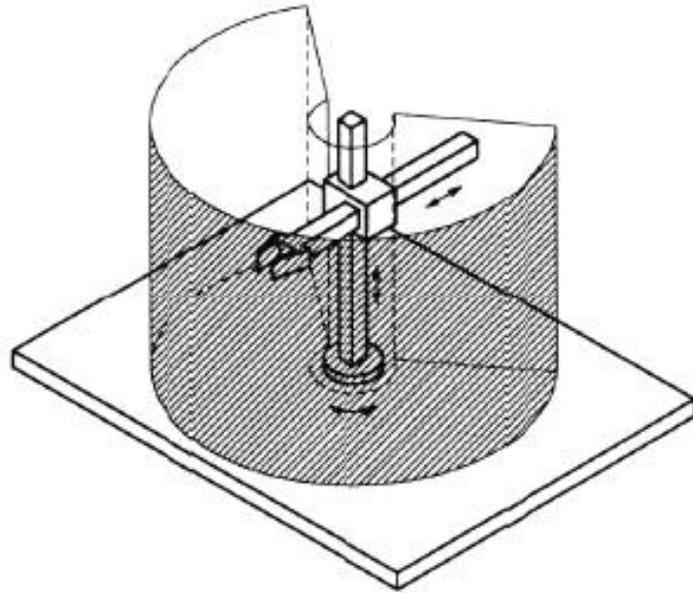
Cartesian



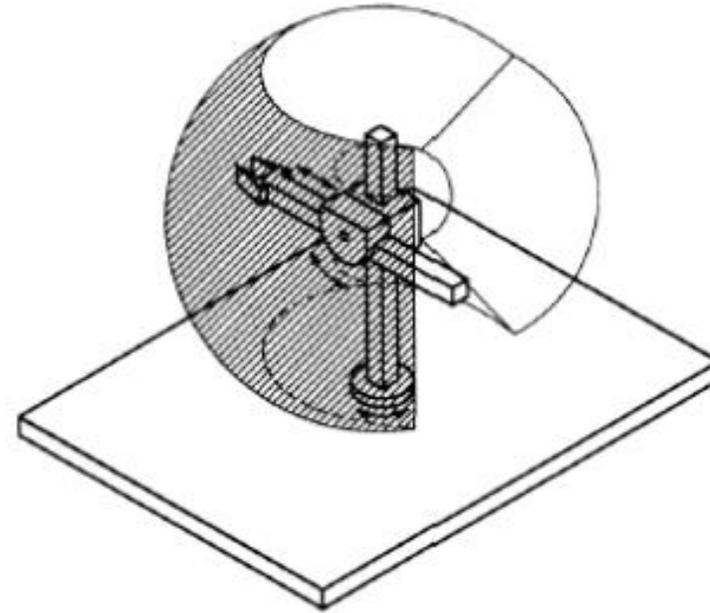
Gantry



# ROBOT CLASSIFICATION WORKSPACE



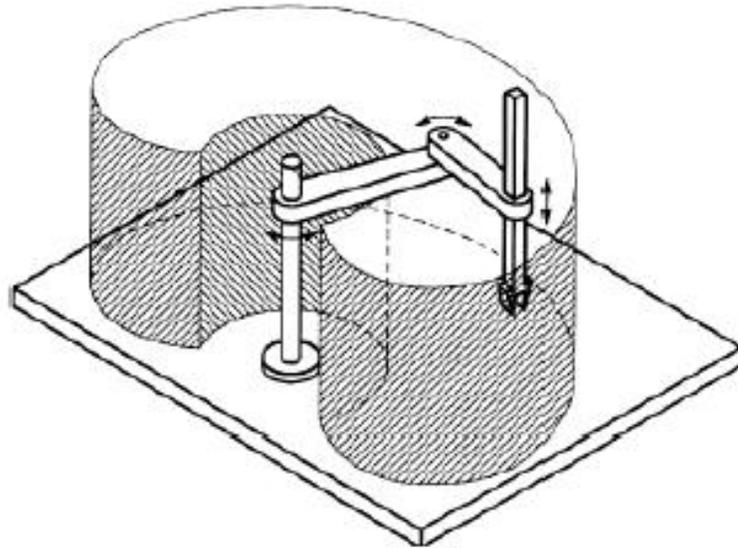
Cylindrical



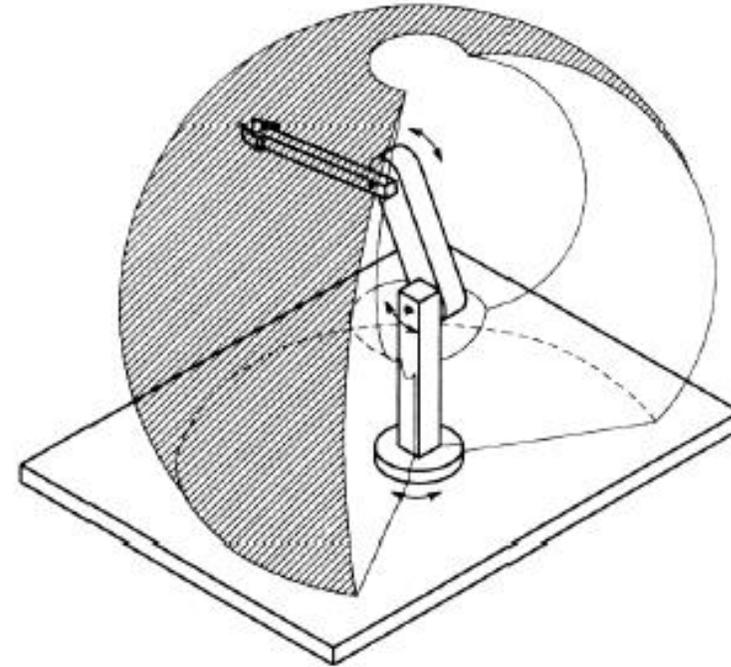
Sphere



# Robot classification workspace



SCARA



Anthropomorphic



# Symbolic representation of robots

## Articulated Configuration (RRR)



Figure 1.4: The Motoman SK16 manipulator.

The articulated manipulator is also called a revolute, or anthropomorphic manipulator.

A common revolute joint design is the parallelogram linkage such as the Motoman SK16, shown in Figure 1.4. In both of these arrangements joint axis  $z_2$  is parallel to  $z_1$  and both  $z_1$  and  $z_2$  are perpendicular to  $z_0$

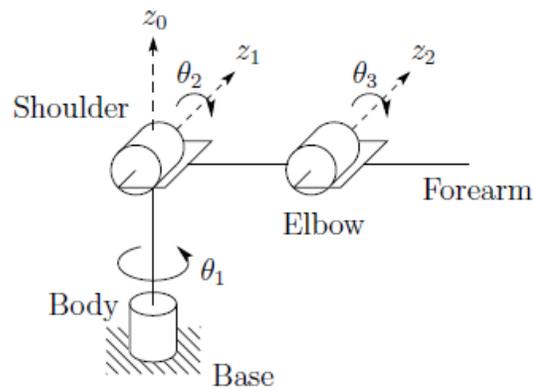


Figure 1.5: Structure of the elbow manipulator.

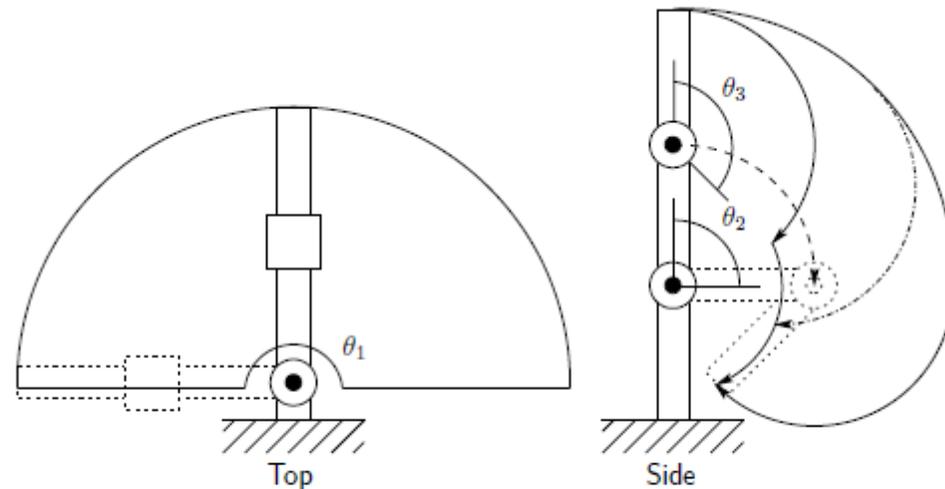


Figure 1.6: Workspace of the elbow manipulator.



# Symbolic representation of robots

## Spherical Configuration (RRP)

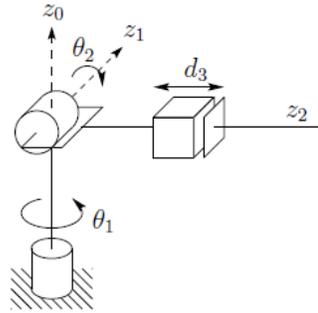
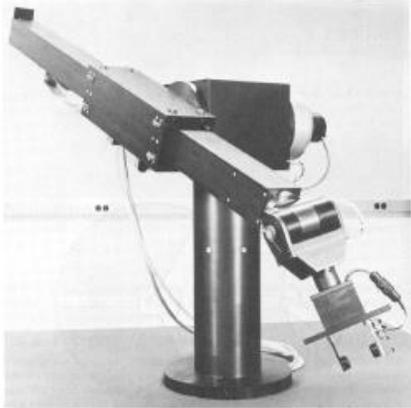


Figure 1.7: The spherical manipulator configuration.

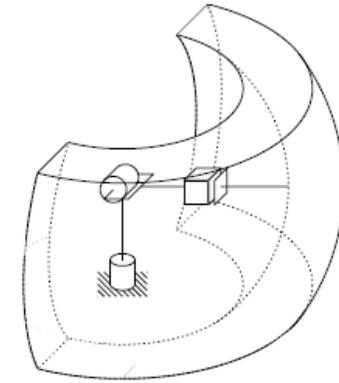


Figure 1.9: Workspace of the spherical manipulator.

By replacing the third or elbow joint in the revolute configuration by a prismatic joint one obtains the spherical configuration shown in Figure 1.7.

The term spherical configuration derives from the fact that the spherical coordinates defining the position of the end-effector with respect to a frame whose origin lies at the intersection of the axes  $z_1$  and  $z_2$  are the same as the first three joint variables.



# Symbolic representation of robots

The so-called SCARA (for Selective Compliant Articulated Robot for Assembly) shown in Figure 1.11 is a popular configuration, which, as its name suggests, is tailored for assembly operations. Although the SCARA has an RRP structure, it is quite different from the spherical configuration in both appearance and in its range of applications.



Figure 1.11: The Epson E2L653S SCARA Robot. Photo Courtesy of Epson.

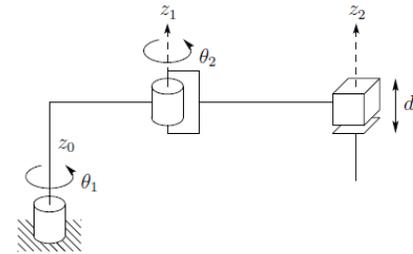


Figure 1.10: The SCARA (Selective Compliant Articulated Robot for Assembly).

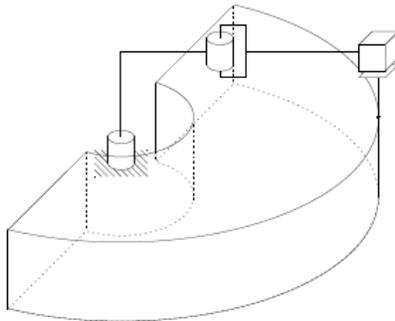
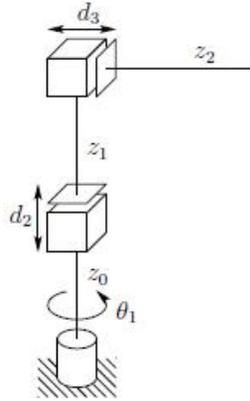


Figure 1.12: Workspace of the SCARA manipulator.



# Symbolic representation of robots

## Cylindrical Configuration (RPP)



The cylindrical configuration is shown in Figure 1.13. The first joint is revolute and produces a rotation about the base, while the second and third joints are prismatic.

As the name suggests, the joint variables are the cylindrical coordinates of the end-effector with respect to the base. A cylindrical robot, the Seiko RT3300, is shown in Figure 1.14, with its workspace shown in Figure 1.15.

Figure 1.13: The cylindrical manipulator configuration.



Figure 1.14: The Seiko RT3300 Robot. Photo courtesy of Seiko.

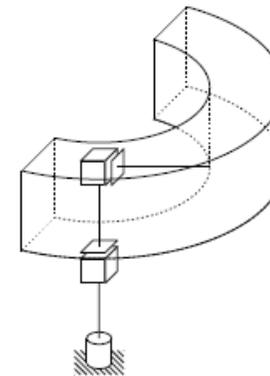


Figure 1.15: Workspace of the cylindrical manipulator.



# Symbolic representation of robots

## Cartesian configuration (PPP)



Figure 1.17: The Epson Cartesian Robot. Photo courtesy of Epson.

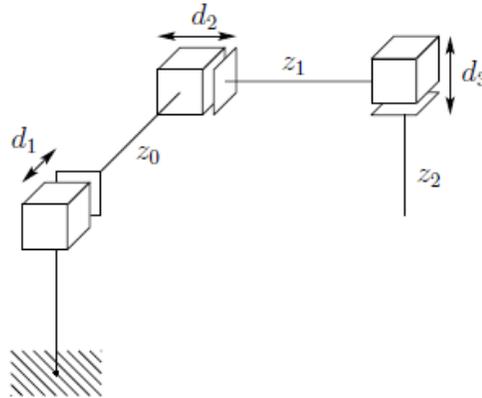


Figure 1.16: The cartesian manipulator configuration.

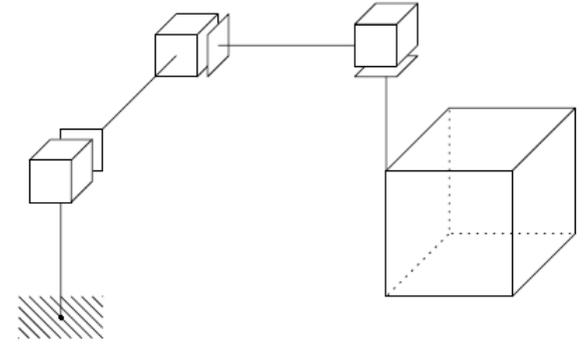


Figure 1.18: Workspace of the cartesian manipulator.

A manipulator whose first three joints are prismatic is known as a cartesian manipulator, shown in Figure 1.16. For the Cartesian manipulator the joint variables are the Cartesian coordinates of the end-effector with respect to the base.

As might be expected the kinematic description of this manipulator is the simplest of all configurations. Cartesian configurations are useful for table-top assembly applications and, as gantry robots, for transfer of material or cargo.

An example of a cartesian robot, from Epson-Seiko, is shown in Figure 1.17.

The workspace of a Cartesian manipulator is shown in Figure 1.18.



# Robotic Systems

A robot manipulator should be viewed as more than just a series of mechanical linkages. The mechanical arm is just one component to an overall Robotic System, shown in Figure 1.20, which consists of the arm, external power source, end-of-arm tooling, external and internal sensors, computer interface, and control computer.

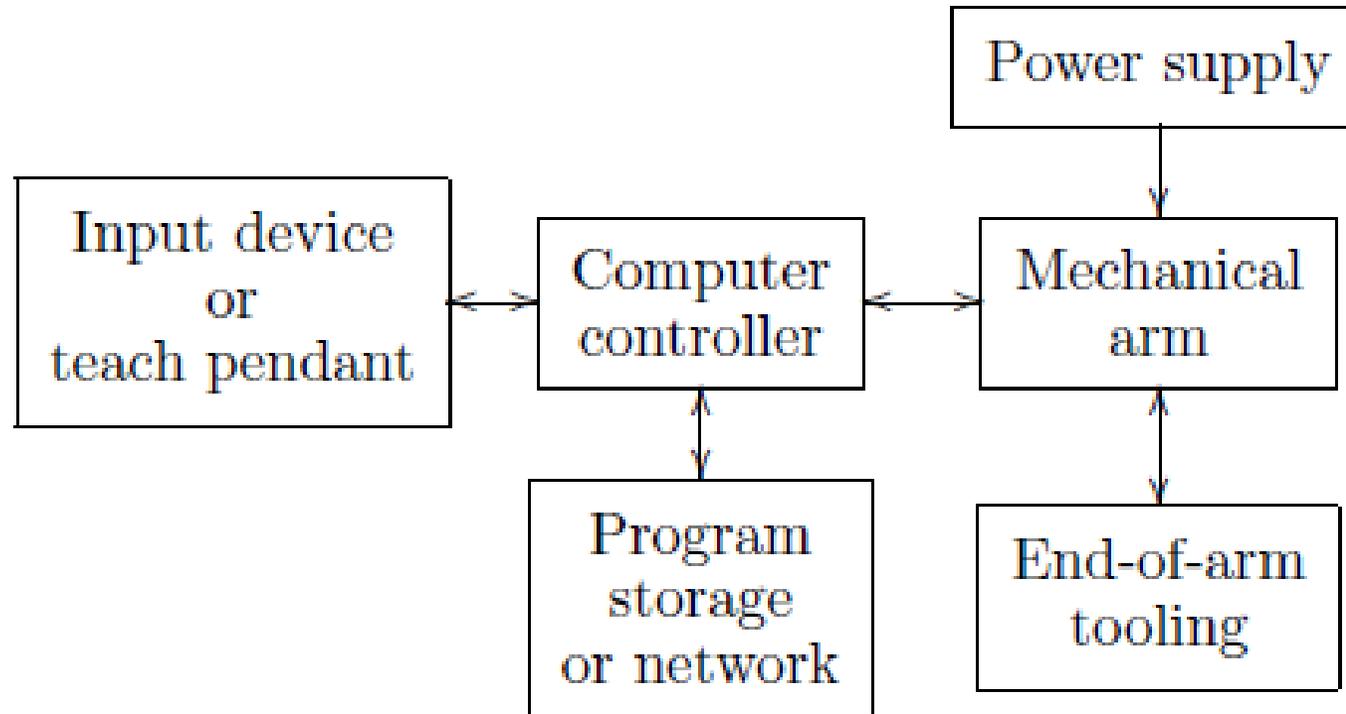


Figure 1.20: Components of a robotic system.



# Robots' Performance

## Accuracy and Repeatability

The **accuracy** of a manipulator is a measure of how close the manipulator can come to a given point within its workspace

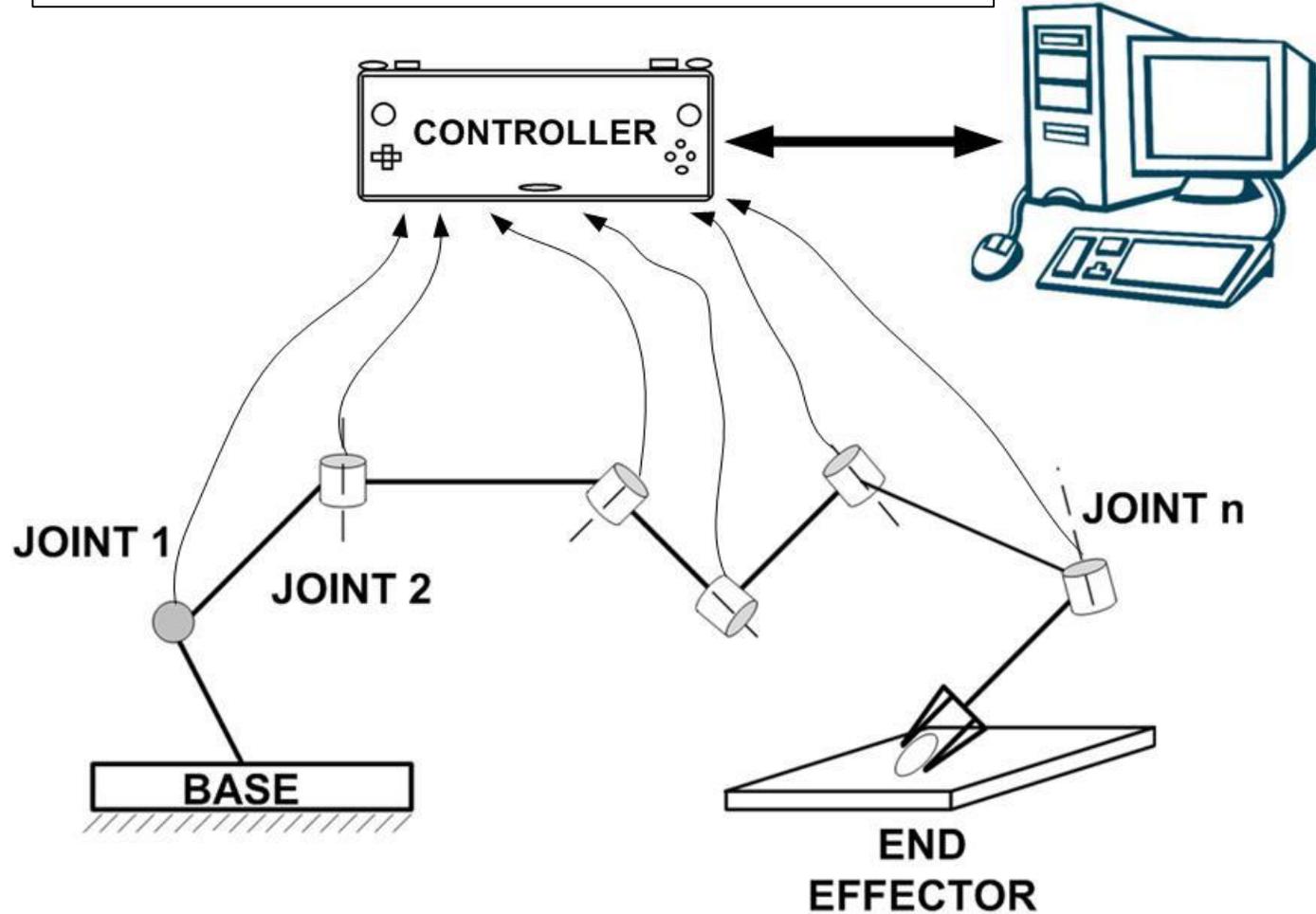
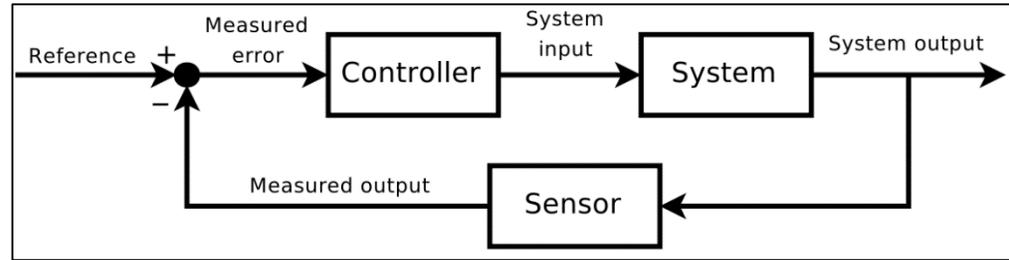
**Repeatability** is a measure of how close a manipulator can return to a previously taught point

**Accuracy** is affected by computational errors, machining accuracy in the construction of the manipulator, flexibility effects such as the bending of the links under gravitational and other loads, gear backlash, and a host of other static and dynamic effects. It is primarily for this reason that robots are designed with extremely high rigidity.

**Repeatability** is affected primarily by the controller resolution. Controller resolution means the smallest increment of motion that the controller can sense. The resolution is computed as the total distance travelled by the tip divided by  $2^n$ , where  $n$  is the number of bits of **encoder** accuracy.

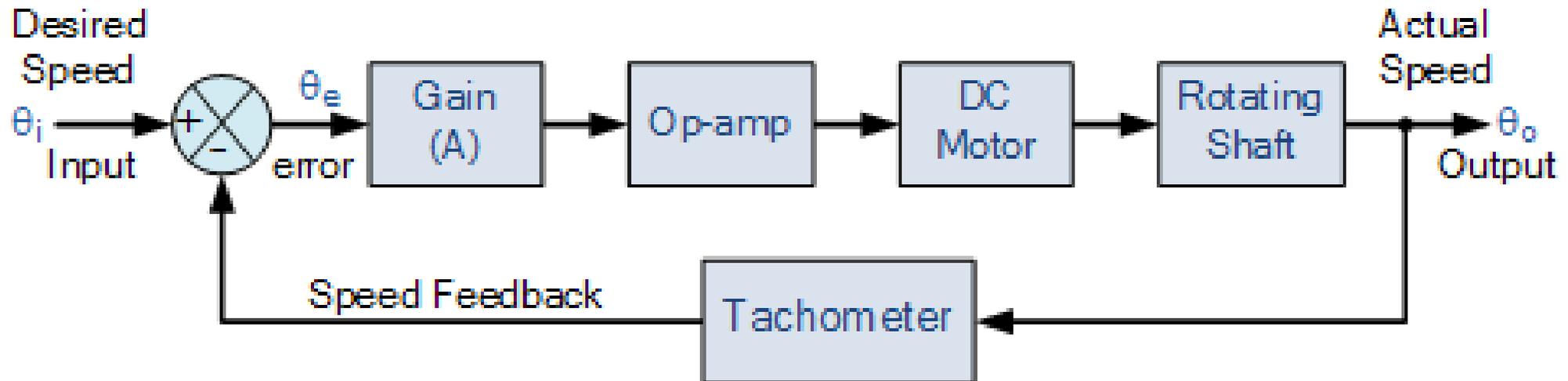
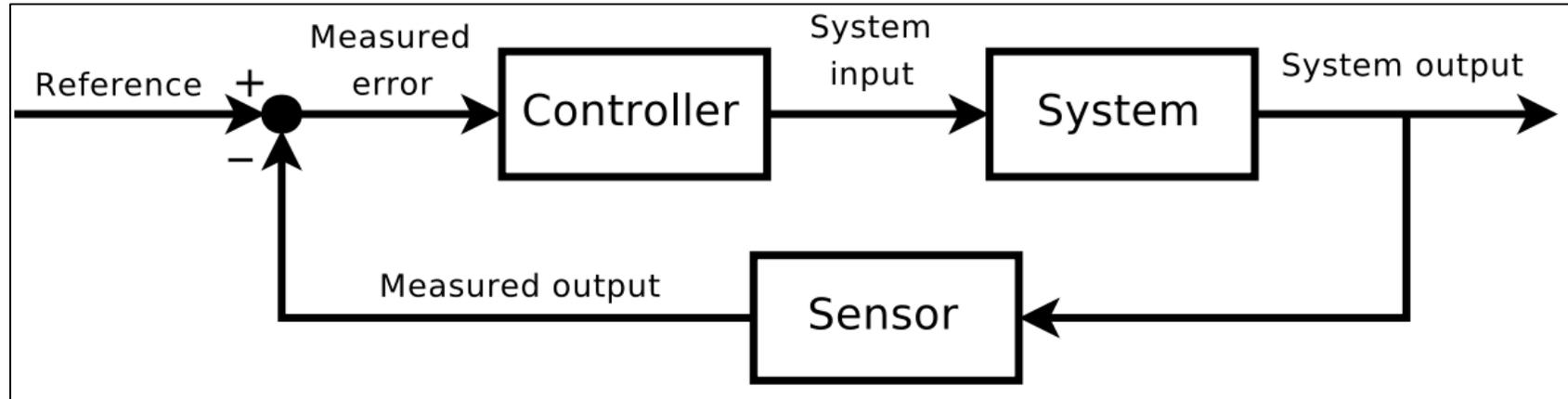


# Sensors and Actuators





# What are we controlling?





# Actuation

## Actuator (in the mechanical sense):

*"A component of a machine that is responsible for moving and controlling a mechanism or system."* ~Wikipedia

*"A mechanical device for moving or controlling something."* ~Merriam-Webster

*"A machine or part of a machine which moves or controls another part in response to an input."* ~Collins

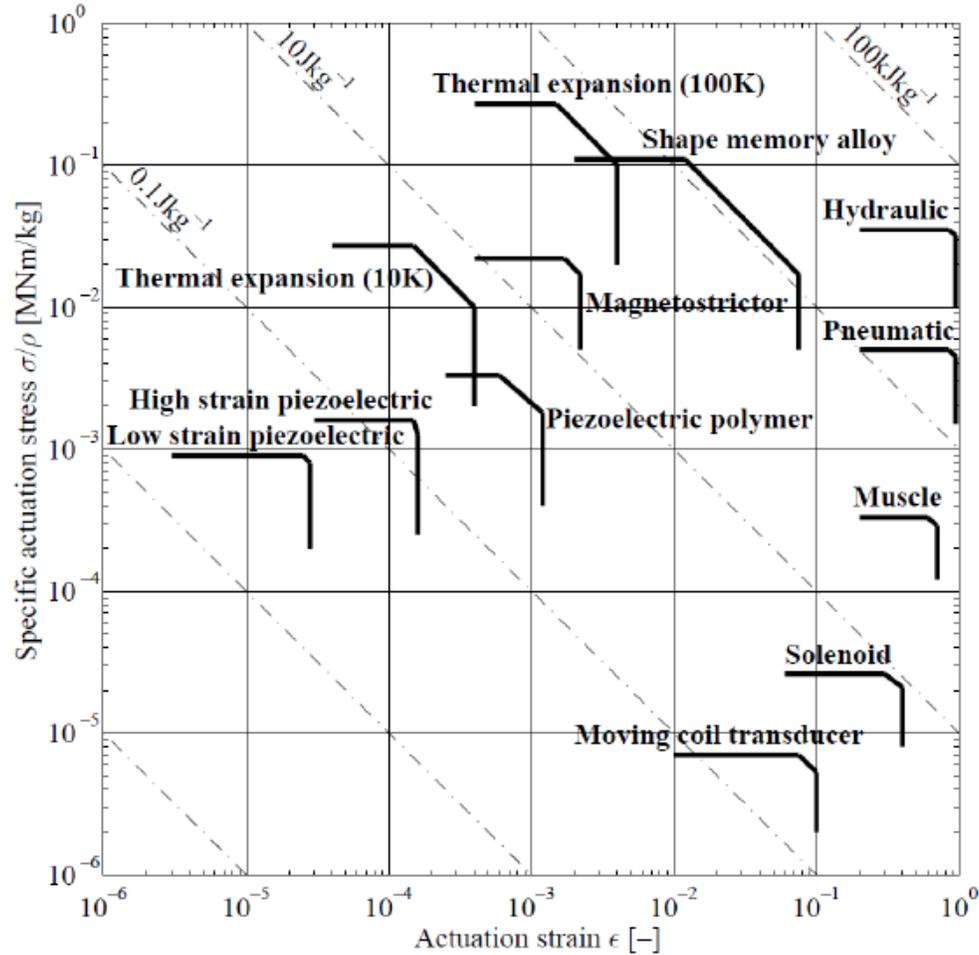


# Actuators type

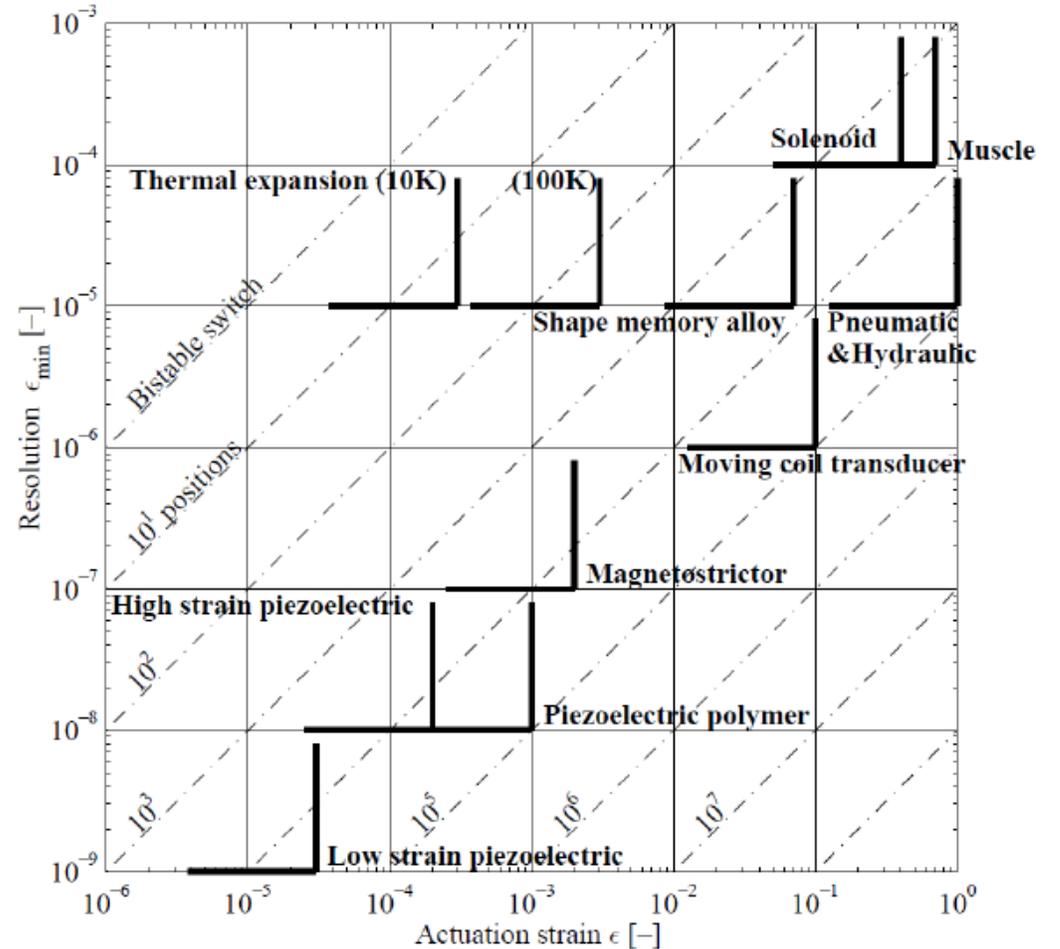
source	actuator family	example
electric (current, voltage)	electromagnetic	Lorentz-type (DC, VCM)
	electrostatic	magnetostrictive
	piezoelectric	comb drive
fluid power (flow, pressure)	hydraulic	vane motor
	pneumatic	cylinder
chemical (potential)	muscle	
thermal (temperature)	shape memory alloy thermal expansion	



# Actuators Comparison



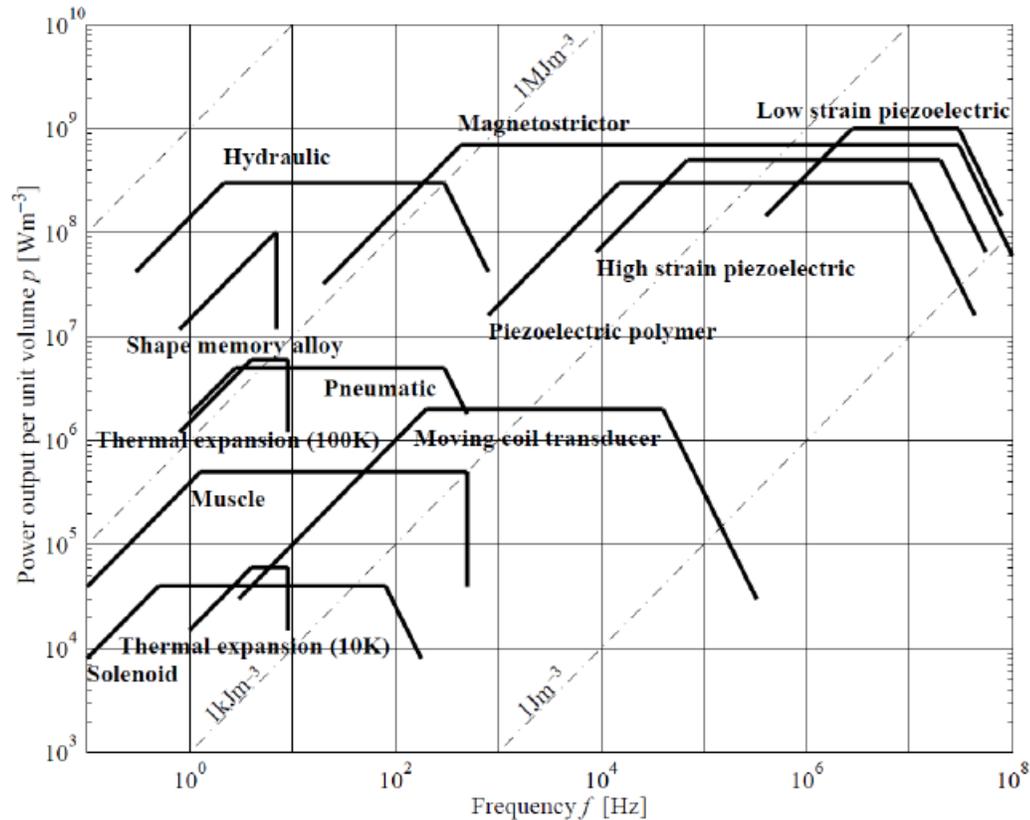
Specific actuation stress,  $\sigma/\rho$ , versus actuation strain,  $\epsilon$ , for various actuators. Heavy lines bound the upper limits of performance.



Strain resolution,  $\epsilon_{min}$ , versus actuation strain,  $\epsilon$ , for various actuators. Heavy lines bound the limits of performance.



# Actuators Comparison



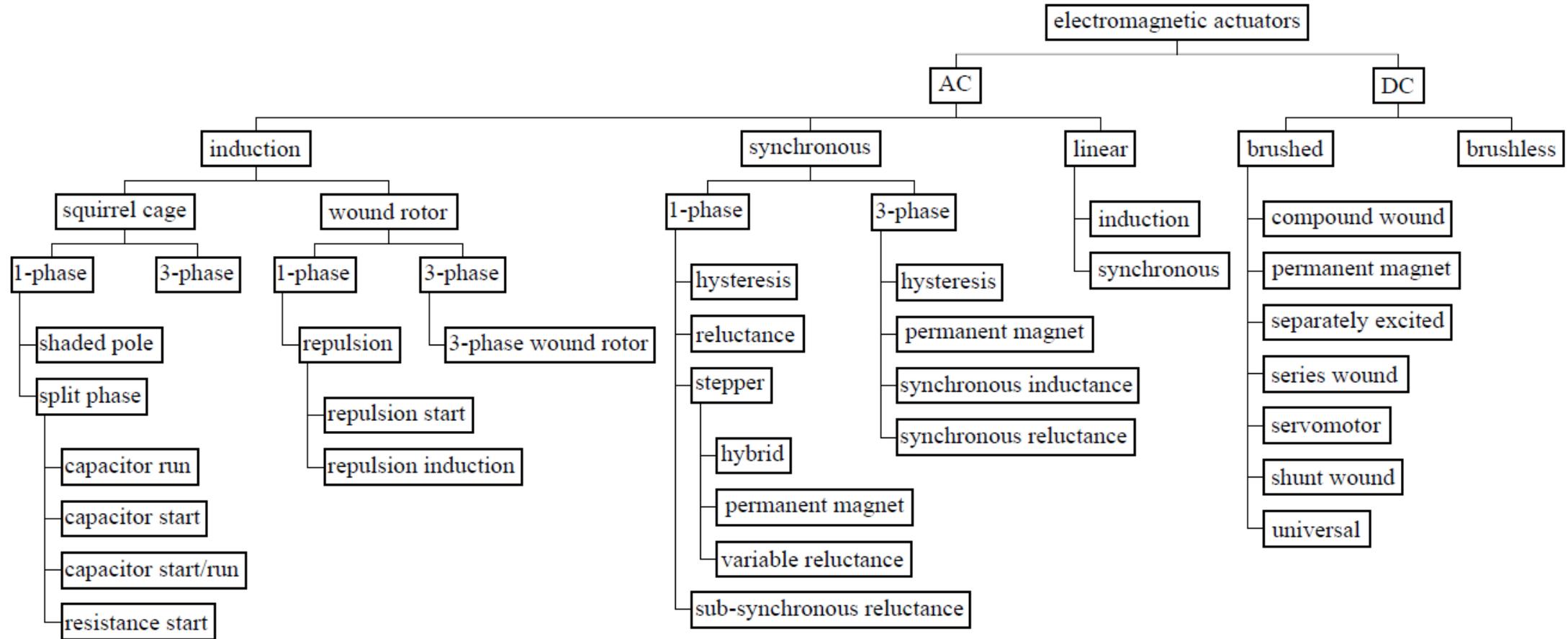
adapted from:

J.E. Huber, N.A. Fleck, and M.F. Ashby, "The selection of mechanical actuators based on performance indices", *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 453, no. 1965, pp. 2185-2205, Oct. 1997

Volumetric power,  $p$ , versus frequency,  $f$ , for various actuators



# Electromagnetic actuators



additional information: <http://www.electrical-knowhow.com/2012/05/classification-of-electric-motors.html>

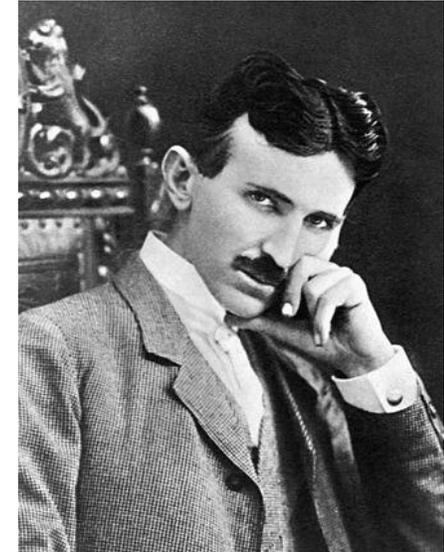


# Electromechanical Actuators



**Galileo Ferraris** (31 October 1847 – 7 February 1897) was an Italian [physicist](#) and [electrical engineer](#), one of the pioneers of [AC power](#) system and an inventor of the three-phase [induction motor](#)

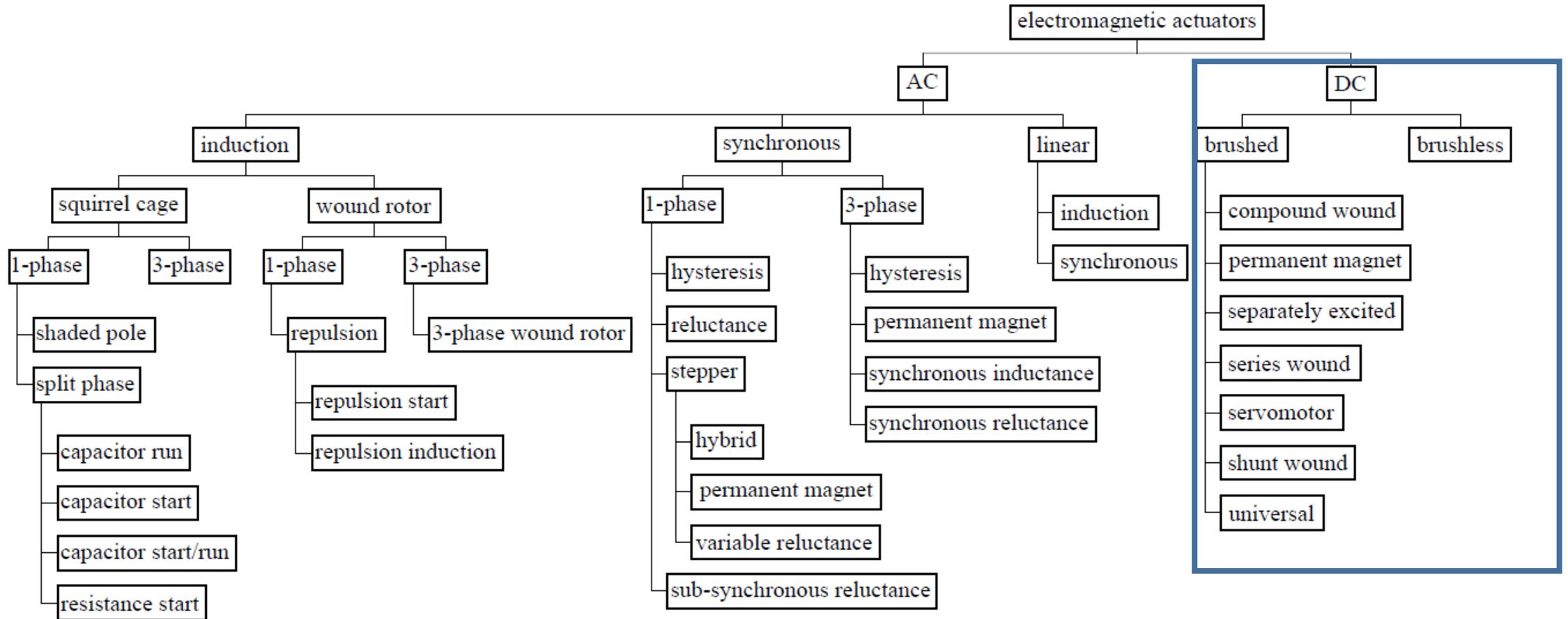
On 11 March 1888, Ferraris published his research in a paper to the Royal Academy of Sciences in [Turin](#) (on May 1888 [Nikola Tesla](#) gained [U.S. Patent 381,968](#), application filed October 12, 1887. Serial Number 252,132)



**Nikola Tesla** (10 July 1856 – 7 January 1943) was a [Serbian-American inventor](#), [electrical engineer](#), [mechanical engineer](#), and [futurist](#) who is best known for his contributions to the design of the modern [alternating current](#) (AC) [electricity supply](#) system. His [alternating current](#) (AC) [induction motor](#) and related [polyphase](#) AC patents, licensed by [Westinghouse Electric](#) in 1888, earned him a considerable amount of money and became the cornerstone of the polyphase system which that company would eventually market.



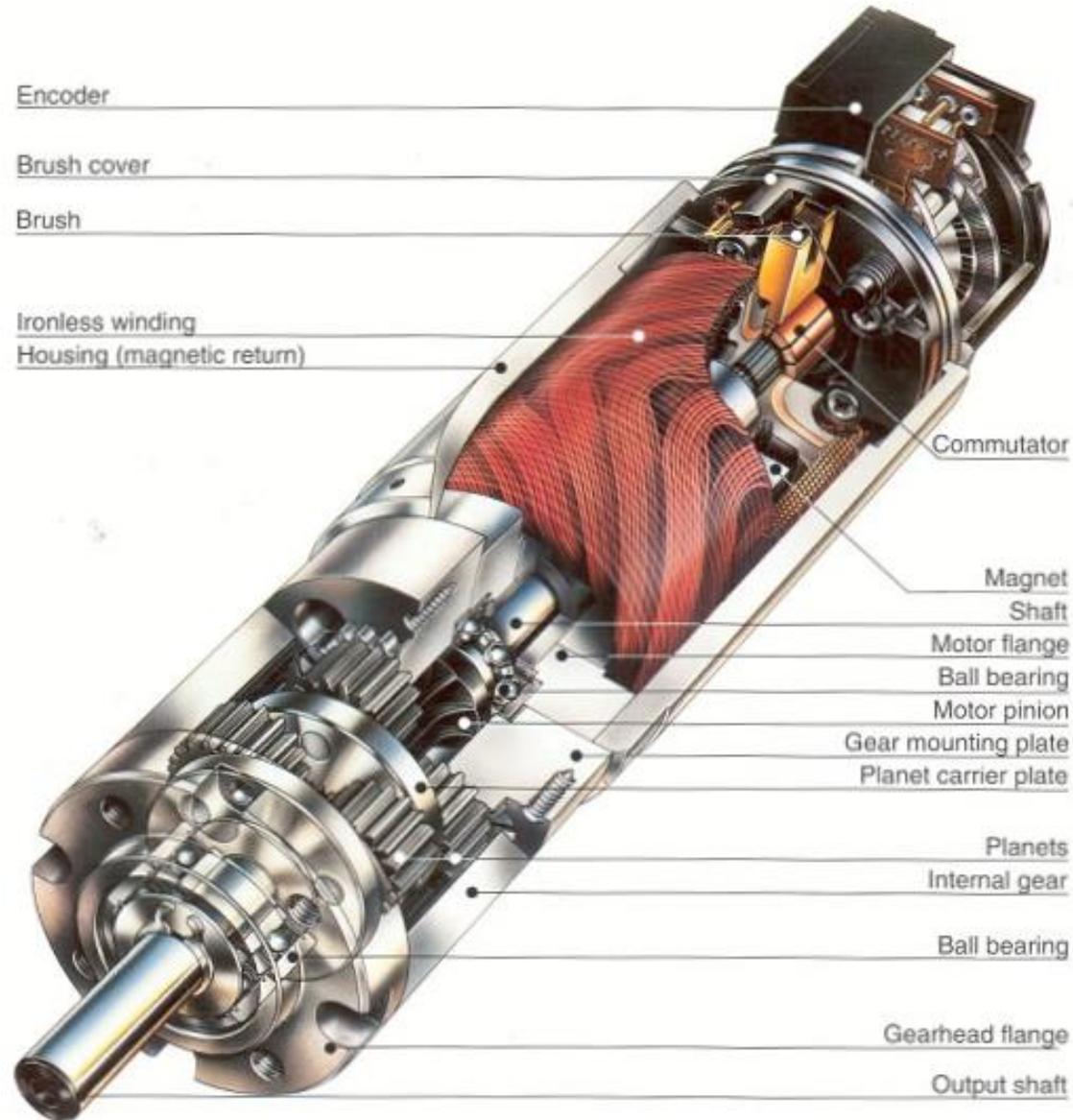
# Electromechanical Actuators



additional information: <http://www.electrical-knowhow.com/2012/05/classification-of-electric-motors.html>

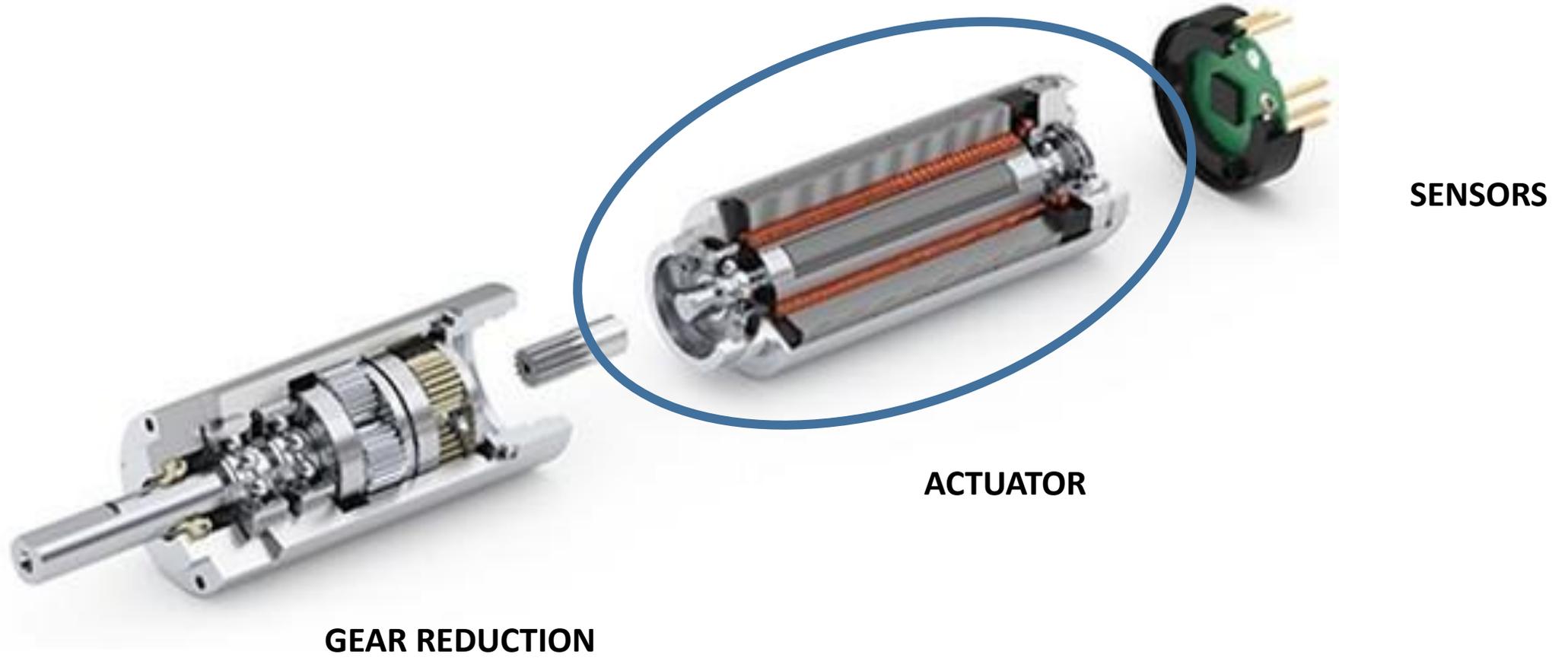


# DC brushed motor





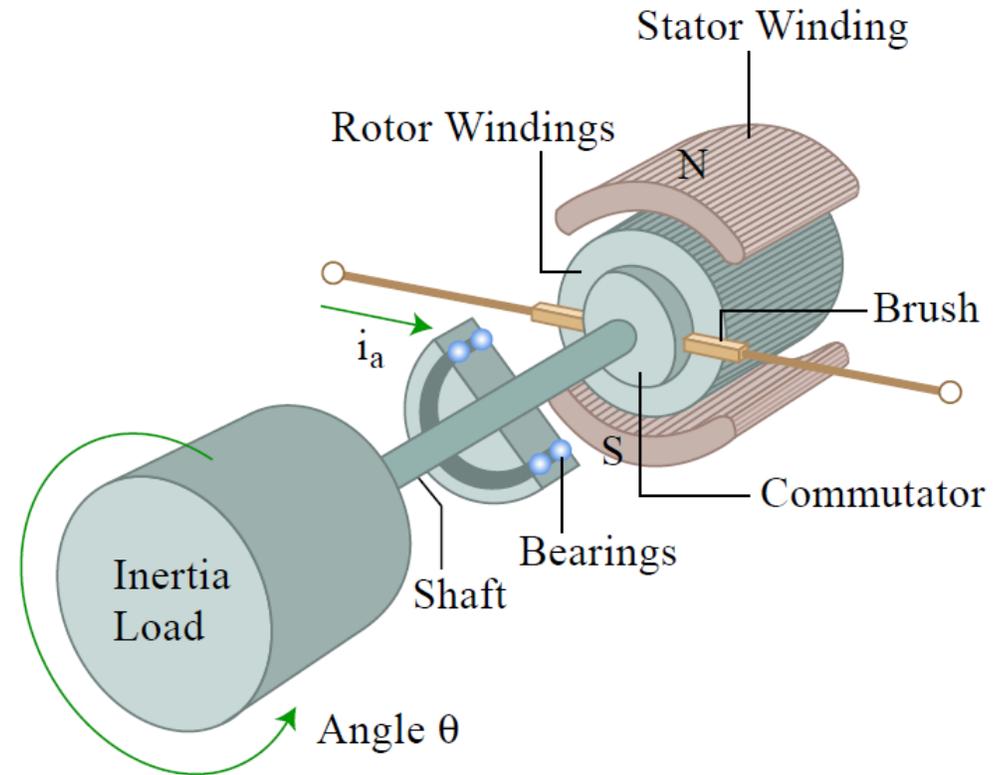
# DC Motor: main parts



# DC Motors

The Figure illustrates the construction of a DC servomotor, consisting of a stator, a rotor, and a commutation mechanism. The **stator** consists of **permanent magnets**, creating a magnetic field in the air gap between the rotor and the stator. The **rotor** has several **windings** arranged symmetrically around the motor shaft. An **electric current** applied to the motor is delivered to individual windings through the **brush-commutation** mechanism, as shown in the figure. As the rotor rotates the polarity of the current flowing to the individual windings is altered. **This allows the rotor to rotate continually.**

- A permanent magnet rotor.
- A stator with a three-, four-, or more phase winding.
- A rotor position sensor.
- An electronic circuit to control the phases of the rotor winding.





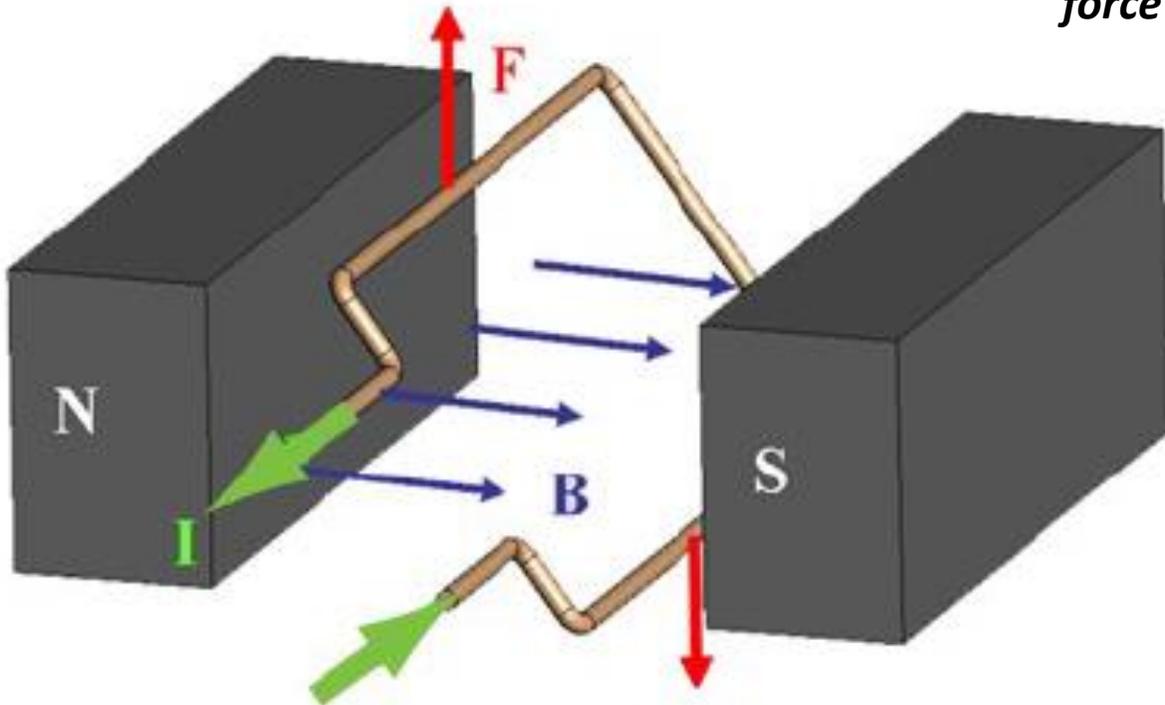
# DC Motors

the Lorentz force law describes the force acting on a moving point charge  $q$  in the presence of electromagnetic fields.

When a wire carrying an electric current is placed in a magnetic field, each of the moving charges, which comprise the current, experiences the Lorentz force, and together they can create a macroscopic force on the wire (sometimes called the **Laplace force**). By combining the Lorentz force law above with the definition of electric current, the following equation results, in the case of a straight, stationary wire:

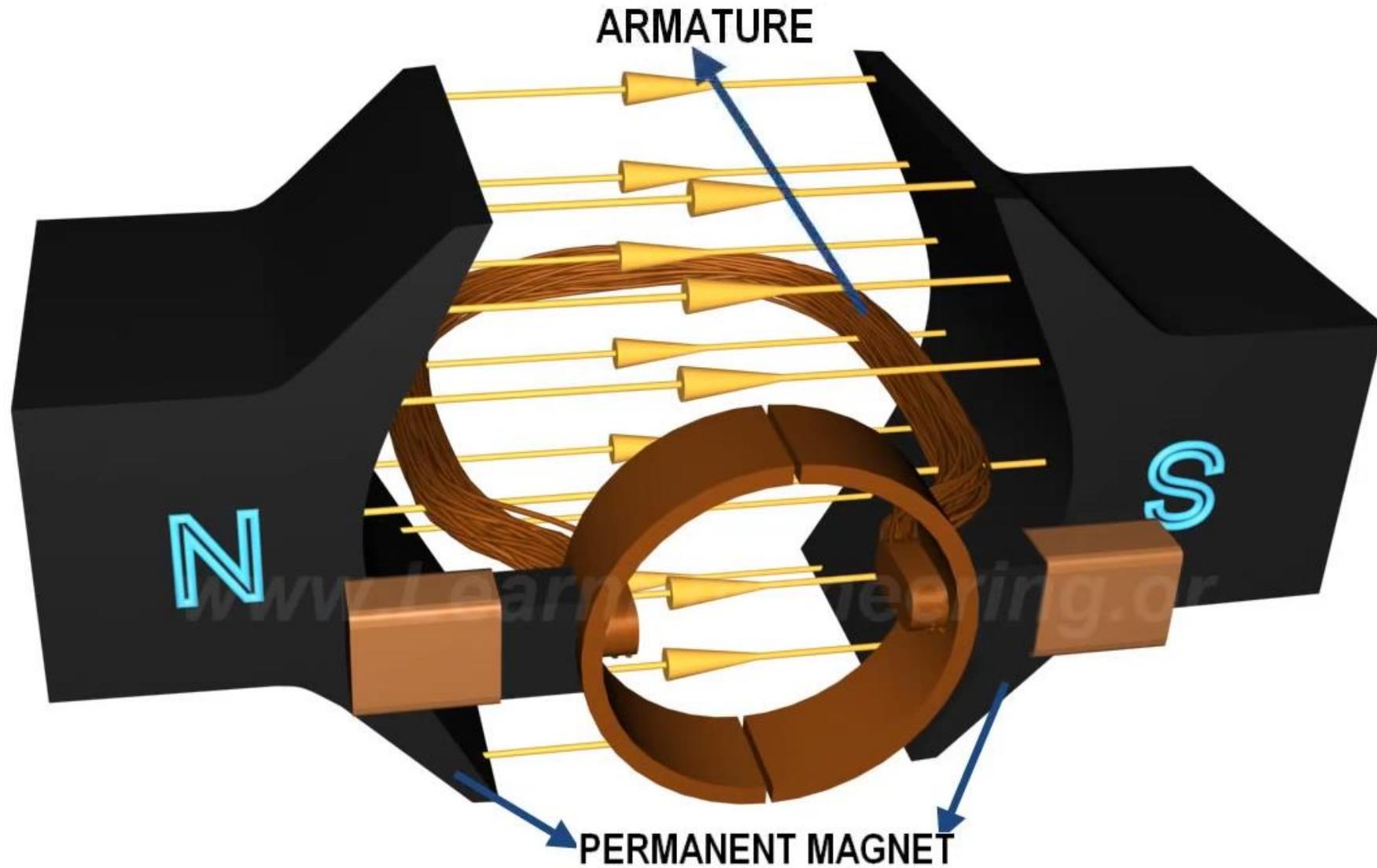
$$\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$$

the Lorentz force is responsible for *motional electromotive force* (or *motional EMF*)



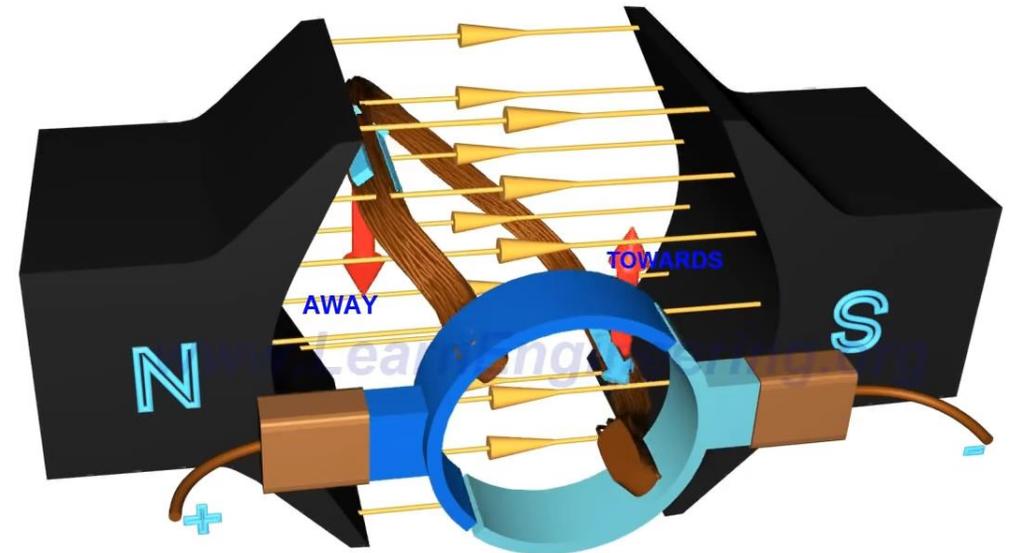
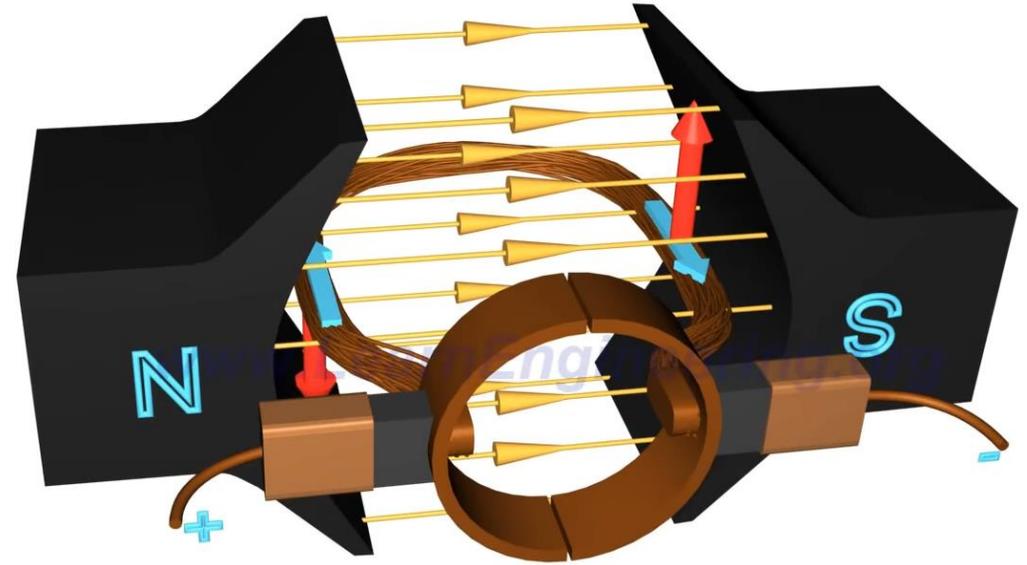
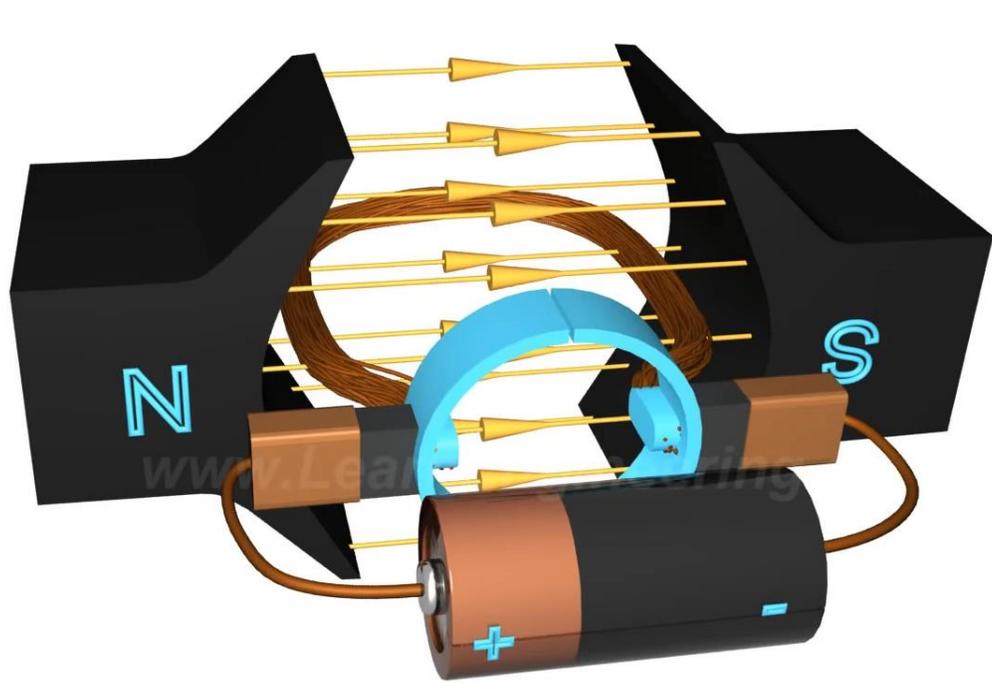


# DC motor structure



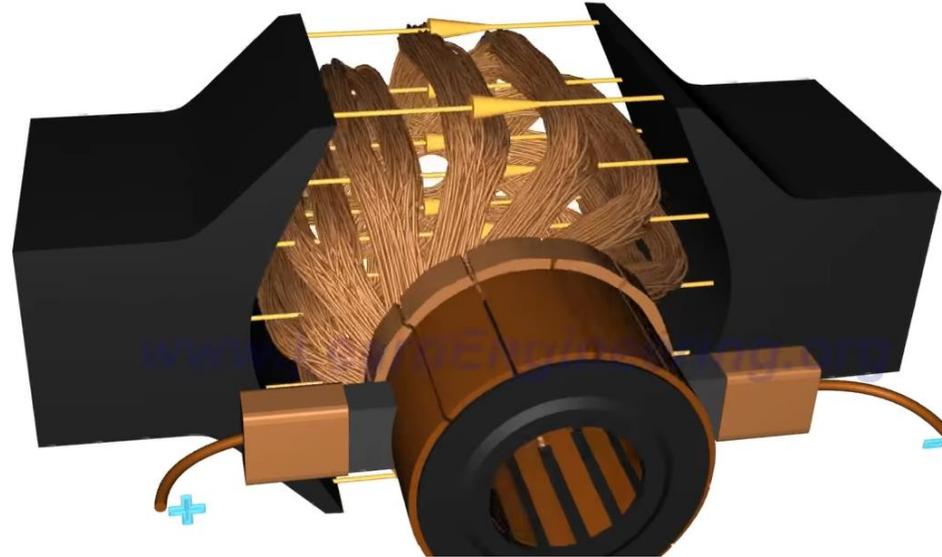
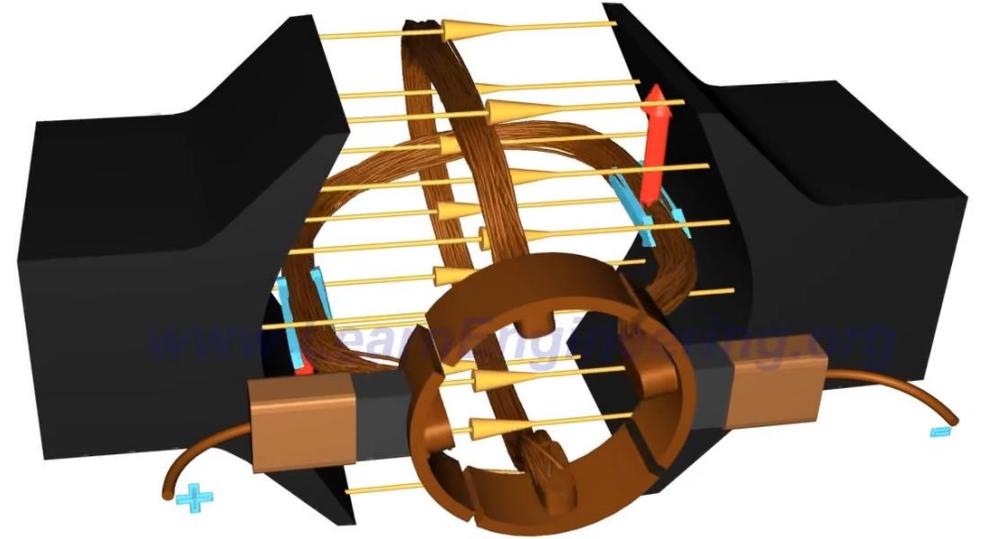
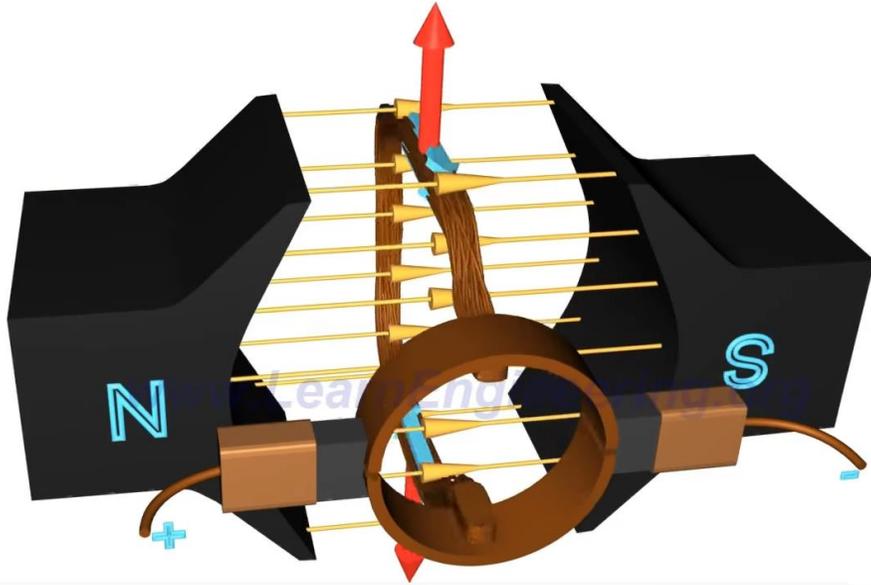


# EFM electromotive force





# More windings for smooth motion



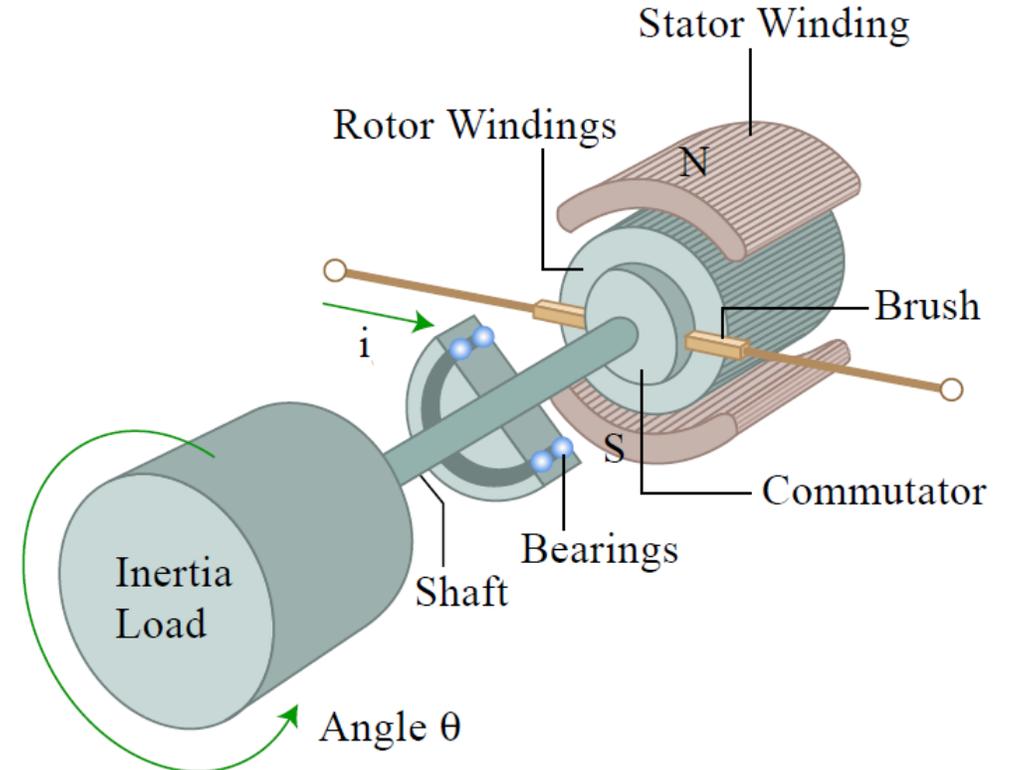


# DC Motor: working principle

Let  $\tau_m$  be the torque created at the air gap, and  $i$  the current flowing to the rotor windings. The torque is in general proportional to the current, and is given by:

$$\tau_m = K_t \cdot i$$

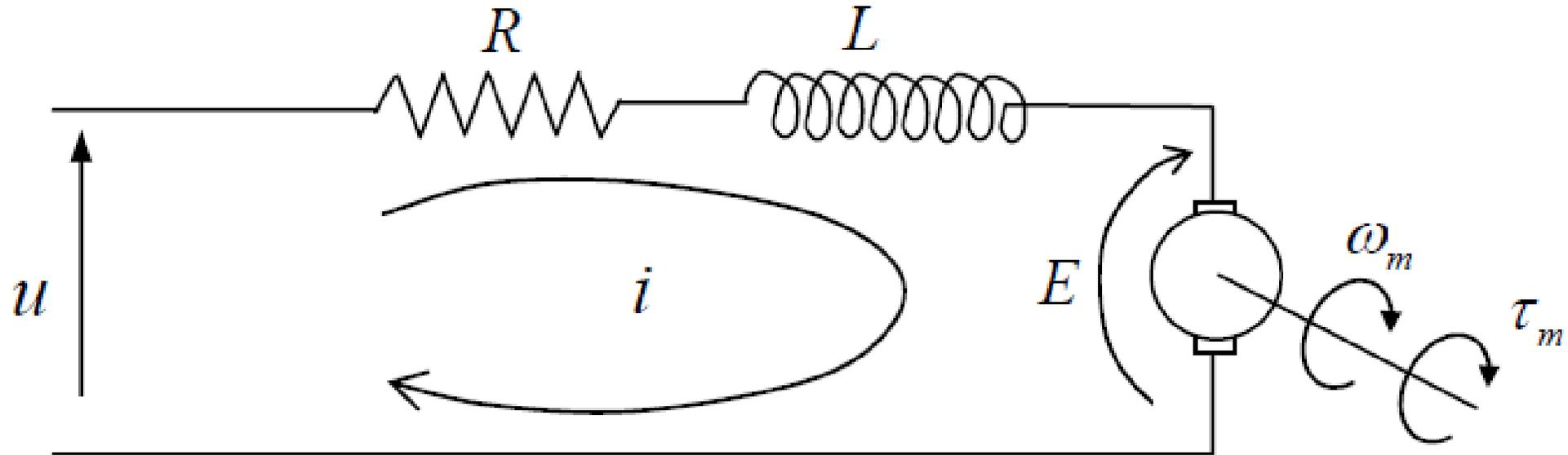
where the proportionality constant  $K_t$  is called the torque constant, one of the key parameters describing the characteristics of a DC motor.



The *torque constant* is determined by the strength of the magnetic field, the number of turns of the windings, the effective area of the air gap, the radius of the rotor, and other parameters associated with materials properties.



# Electrical representation of DC Motor





# DC Motor: working principle

$$\underline{\tau_m = K_t \cdot i} \quad (1)$$

Let  $E$  be the voltage applied to the idealized transducer. The electric power is then given by  $E \cdot i$ , which must be equivalent to the mechanical power:

$$\underline{P_{in} = E \cdot i = \tau_m \cdot \omega_m} \quad (2)$$

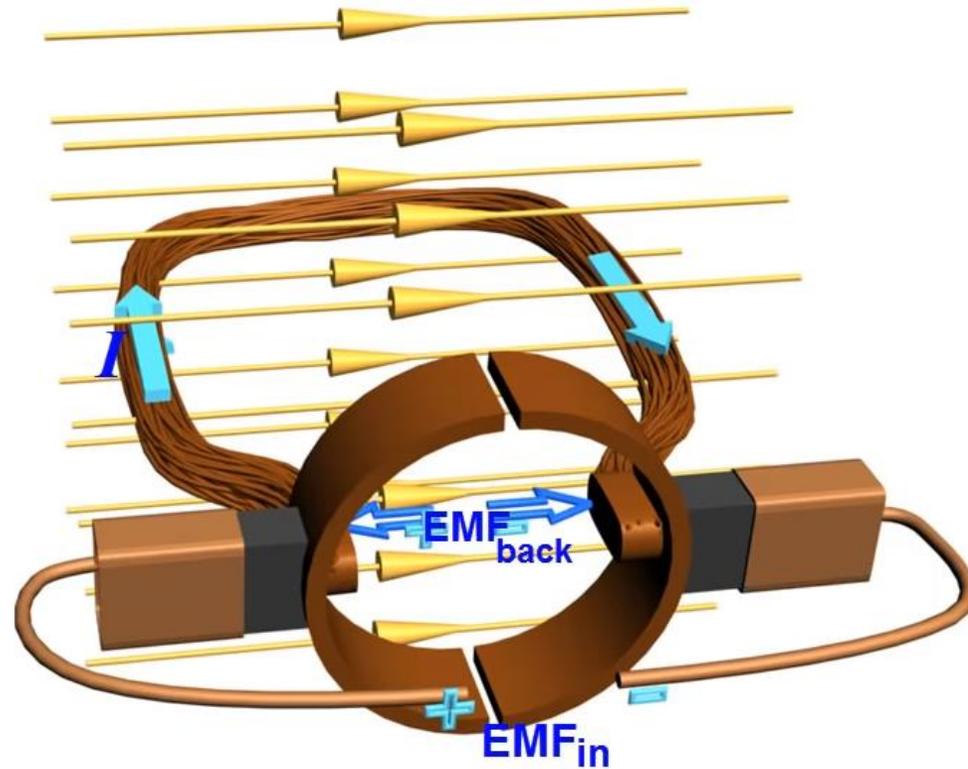
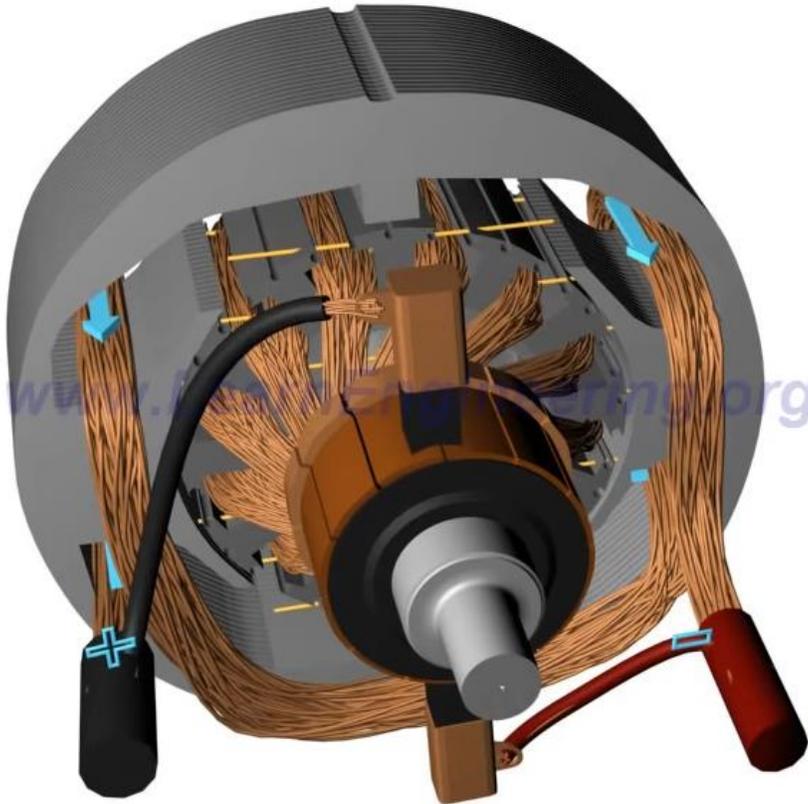
where  $\omega_m$  is the angular velocity of the motor rotor. Substituting eq.(1) into eq.(2) and dividing both sides by  $i$  yield the second fundamental relationship of a DC motor:

$$\underline{E = K_t \omega_m} \quad (3)$$



# Back EMF

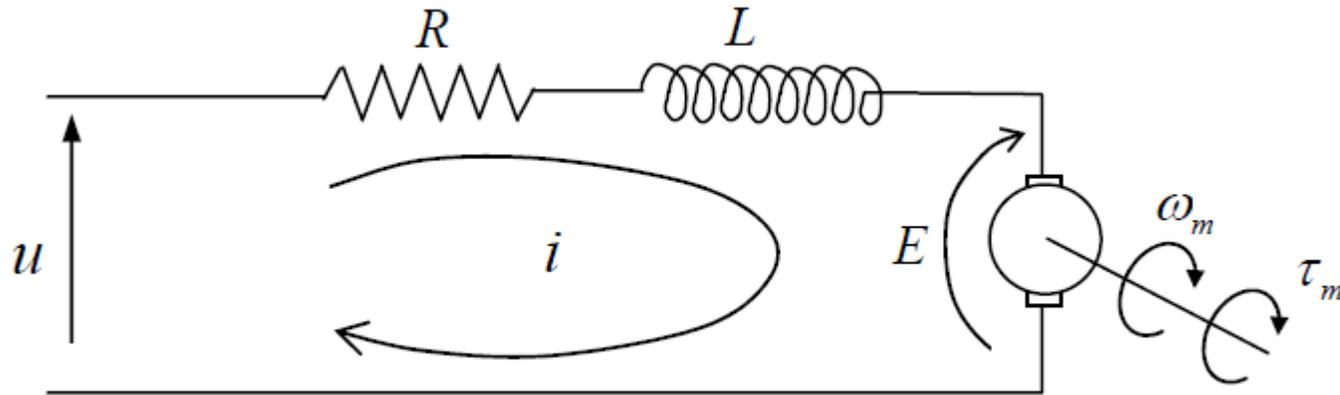
It is an internal voltage which generates inside the windings and opposes to the motion by reducing the circulating current  $I$ .



$$I = \frac{EMF_{in} - EMF_{back}}{R}$$



# DC Motor: working principle



The actual DC motor is not a loss-less transducer, having resistance at the rotor windings and the commutation mechanism. Furthermore, windings may exhibit some inductance, which stores energy. Figure shows the schematic of the electric circuit, including the windings resistance  $R$  and inductance  $L$ . From the figure,

$$\underline{u = R \cdot i + L \frac{di}{dt} + E} \quad \text{where } u \text{ is the voltage applied to the armature of the motor.} \quad (4)$$



# DC Motor: working principle

Combining eqs.(1), (3) and (4), we can obtain the actual relationship among the applied voltage  $u$ , the rotor angular velocity  $\omega_m$ , and the motor torque  $\tau_m$ .

$$\frac{K_t}{R}u = \tau_m + T_e \frac{d\tau_m}{dt} + \frac{K_t^2}{R}\omega_m \quad (5)$$

where time constant  $T_e = \frac{L}{R}$ , called the motor reactance, is often negligibly small. Neglecting this second term, the above equation reduces to an algebraic relationship:

$$\tau_m = \frac{K_t}{R}u - \frac{K_t^2}{R}\omega_m \quad (6)$$



# DC Motor: working principle

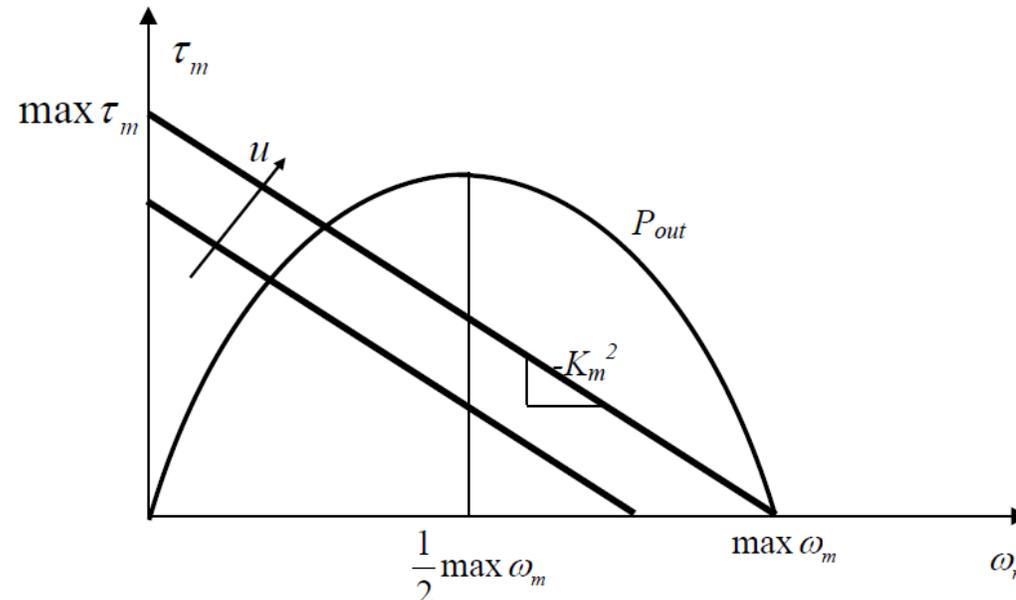
$$\tau_m = \frac{K_t}{R} u - \frac{K_t^2}{R} \omega_m \quad (6)$$

This is called the torque-speed characteristic. Note that the motor torque increases in proportion to the applied voltage, but the net torque reduces as the angular velocity increases. Figure illustrates the torque-speed characteristics. The negative slope of the straight lines,  $-K_t^2/R$ , implies that the voltage-controlled DC motor has an inherent damping in its mechanical behavior.

The power dissipated in the DC motor is given by

$$P_{dis} = R \cdot i^2 = \frac{R}{K_t^2} \tau_m^2 \quad (7)$$

$$P_{out} = \omega_m \tau_m$$



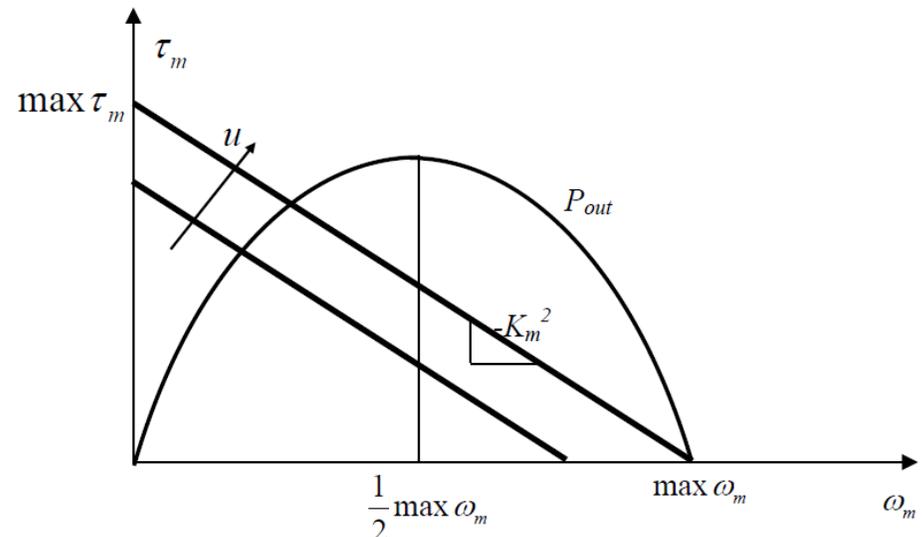


# DC Motor: working principle

from eq.(1). Taking the square root of both sides yields

$$\sqrt{P_{dis}} = \frac{\tau_m}{K_m}, \quad K_m = \frac{K_t}{\sqrt{R}} \quad (8)$$

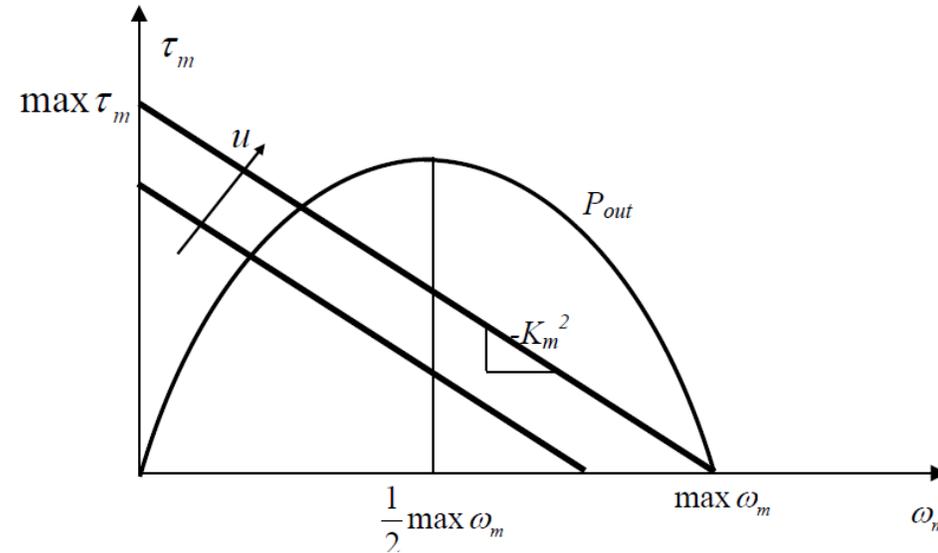
where the parameter  $K_m$  is called the motor constant. The motor constant represents how effectively electric power is converted to torque. The larger the motor constant becomes, the larger the output torque is generated with less power dissipation. A DC motor with more powerful magnets, thicker winding wires, and a larger rotor diameter has a larger motor constant. A motor with a larger motor constant, however, has a larger damping, as the negative slope of the torque-speed characteristics becomes steeper, as illustrated in Figure





Taking into account the internal power dissipation, the net output power of the DC motor is given by

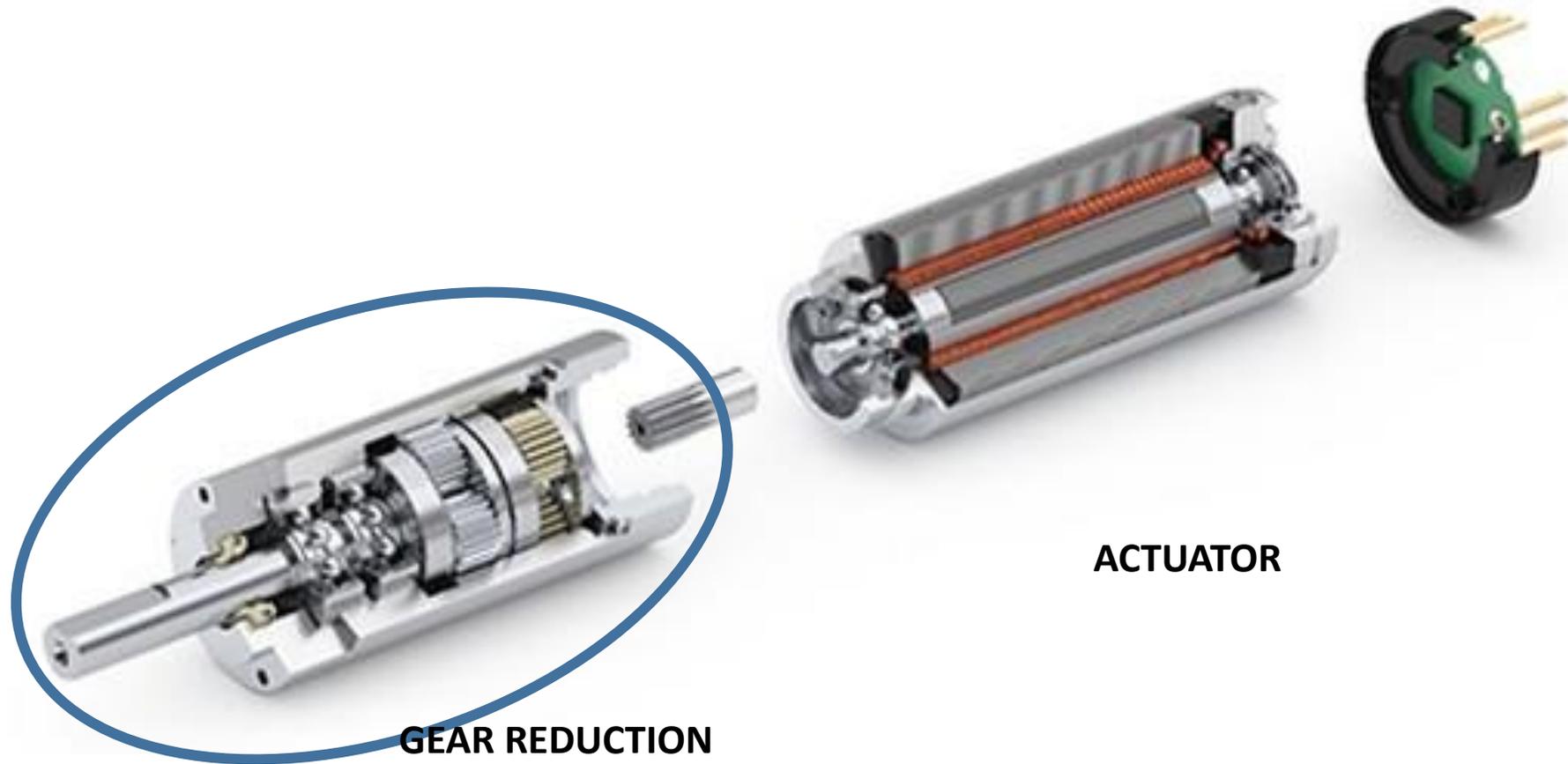
$$P_{out} = \tau_m \cdot \omega_m = \left( \frac{K_t}{R} u - K_m^2 \omega_m \right) \omega_m \quad (9)$$



This net output power is a parabolic function of the angular velocity, as illustrated in Figure. It should be noted that the net output power becomes maximum in the middle point of the velocity axis, i.e. 50 % of the maximum angular velocity for a given armature voltage  $u$ . This implies that the motor is operated most effectively at 50 % of the maximum speed. As the speed departs from this middle point, the net output power decreases, and it vanishes at the zero speed as well as at the maximum speed. Therefore, it is important to select the motor and gearing combination so that the maximum of power transfer be achieved.



# DC Motor: Gears reduction



**GEAR REDUCTION**

**ACTUATOR**

**SENSORS**

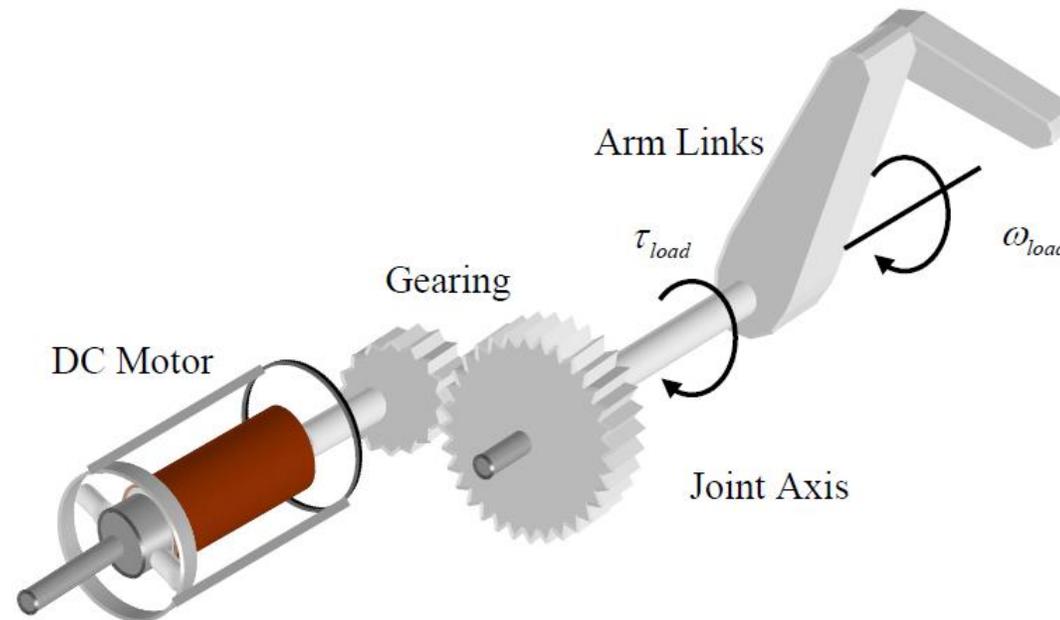


# Dynamics of Single-Axis Drive Systems

DC motors and other types of actuators are used to drive individual axes of a robotic system. Figure shows a schematic diagram of a single-axis drive system consisting of a DC motor, a gear head, and arm links. An electric motor, such as a DC motor, produces a relatively small torque and rotates at a high speed, whereas a robotic joint axis in general rotates slowly, and needs a high torque to bear the load. In other words, the impedance of the actuator:

$$Z_m = \frac{\text{torque}}{\text{angular velocity}} = \frac{\tau_m}{\omega_m}$$

is much smaller than that of the load. (1)



To fill the gap we need a gear reducer, as shown in Figure .

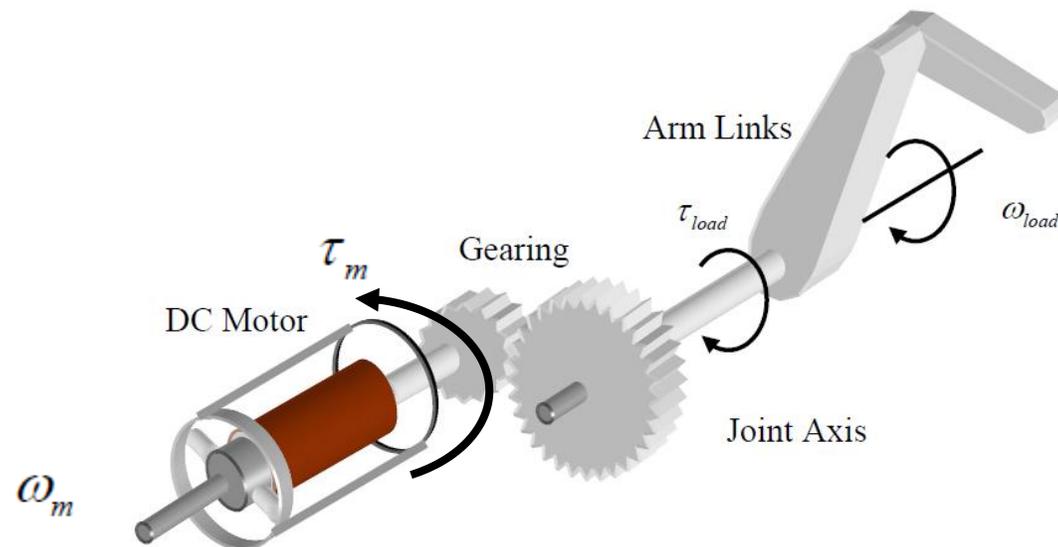


# Dynamics of Single-Axis Drive Systems

Let  $r > 1$  be a gear reduction ratio (If  $d_1$  and  $d_2$  are diameters of the two gears, the gear reduction ratio is  $r = d_2 / d_1$ ). The torque and angular velocity are changed to:

$$\tau_{load} = r \cdot \tau_m, \quad \omega_{load} = \frac{1}{r} \omega_m \quad (2)$$

where  $\tau_{load}$  and  $\omega_{load}$  are the torque and angular velocity at the joint axis, as shown in the figure.





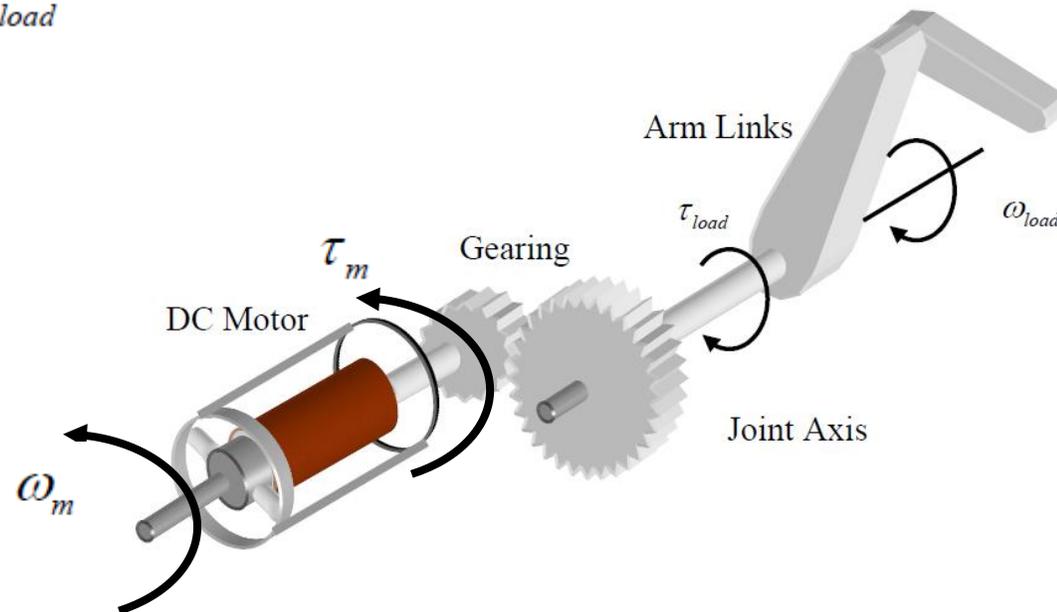
# Impedance of the motor and load

Note that the gear reducer of gear ratio  $r$  increases the impedance  $r^2$  times larger than that of the motor axis  $Z_m$ :

$$Z_{load} = r^2 \cdot Z_m \quad (3)$$

Let  $I_m$  be the inertia of the motor rotor. From the free body diagram of the motor rotor,

$$I_m \dot{\omega}_m = \tau_m - \frac{1}{r} \tau_{load} \quad (4)$$





Let  $I_l$  be the inertia of the arm link about the joint axis, and  $b$  the damping coefficient of the bearings supporting the joint axis. Considering the free body diagram of the arm link and joint axis yields

$$I_l \dot{\omega}_{load} = \tau_{load} - b \omega_{load} \quad (5)$$

Eliminating  $\tau_{load}$  from the above two equations and using eq.  $\tau_m = \frac{K_t}{R} u - \frac{K_t^2}{R} \omega_m$  and (2) yields

$$\underline{I \dot{\omega}_{load} + B \omega_{load} = k \cdot u} \quad (6)$$

where  $I$ ,  $B$ ,  $k$  are the effective inertia, damping, and input gain reflected to the joint axis:

$$I = I_l + r^2 I_m \quad (7)$$

$$B = b + r^2 K_m^2 \quad (8)$$

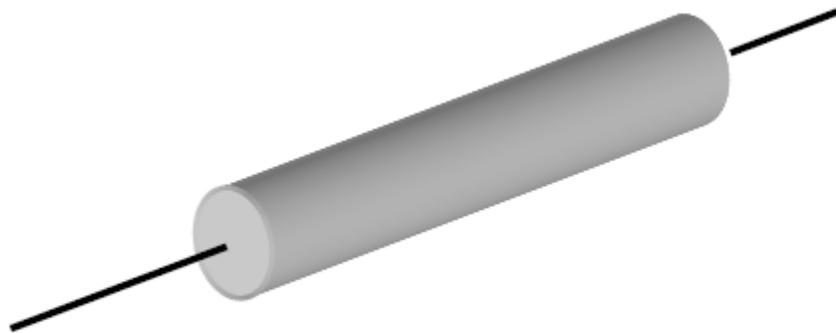
$$k = r \frac{K_t}{R} \quad (9)$$



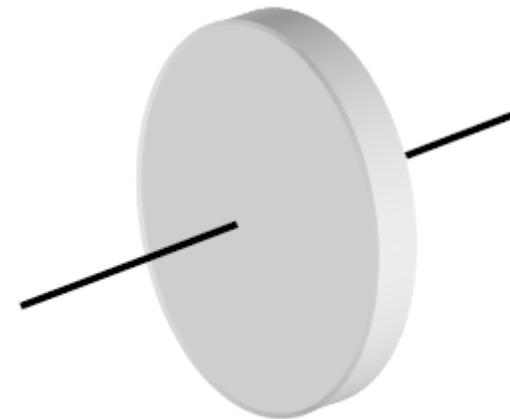
$$I = I_l + r^2 I_m$$

Note that the effective inertia of the motor rotor is  $r^2$  times larger than the original value  $I_m$  when reflected to the joint axis. Likewise, the motor constant becomes  $r^2$  times larger when reflected to the joint axis. The gear ratio of a robotic system is typically  $20 \sim 100$ , which means that the effective inertia and damping becomes  $400 \sim 10,000$  times larger than those of the motor itself.

For fast dynamic response, the inertia of the motor rotor must be small. This is a crucial requirement as the gear ratio gets larger, like robotics applications.



(a) Long and slender



(b) Pancake



# Type of motors





# Power Electronics

Performance of servomotors used for robotics applications highly depends on electric power amplifiers and control electronics, broadly termed power electronics. Power electronics has shown rapid progress in the last two decades, as semiconductors became faster, more powerful, and more efficient. In this section we will briefly summarize power electronics relevant to robotic system development.



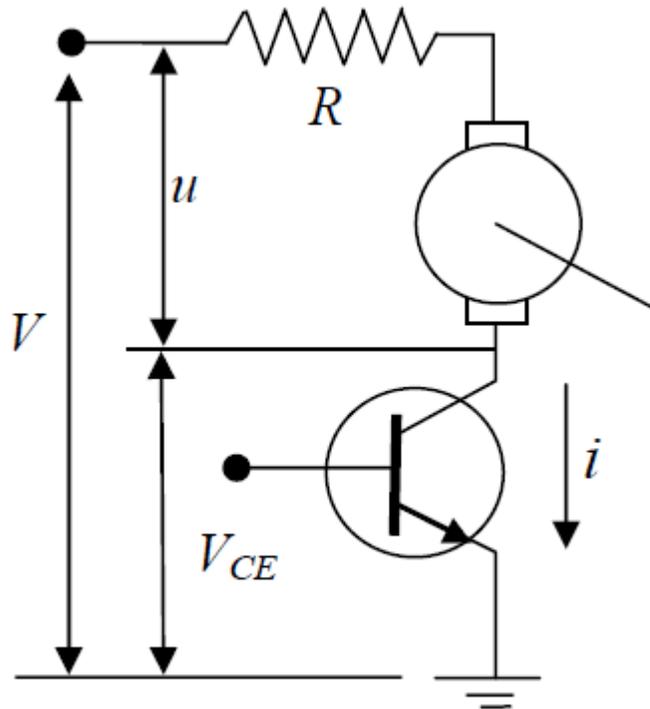


# Pulse width modulation (PWM)

- In many robotics applications, actuators must be controlled precisely so that desired motions of arms and legs may be attained.
- This requires a power amplifier to drive a desired level of voltage (or current indirectly) to the motor armature, as discussed in the previous section.
- Use of a linear amplifier (like an operational amplifier), however, is power-inefficient and impractical, since it entails a large amount of power loss.

# Analog Amplifier (Why not)

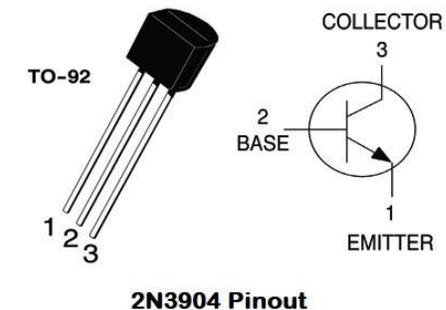
Consider a simple circuit consisting of a single transistor for controlling the armature voltage, as shown in Figure



- Let  $V$  be the supply voltage connected to one end of the motor armature (winding).
- The other end of the armature is connected to the collector of the transistor.
- As the base voltage varies the emitter-collector voltage varies, and thereby the voltage drop across the motor armature, denoted  $u$  in the figure, varies accordingly.
- Let  $i$  be the collector current flowing through the transistor.
- Then the power loss that is dissipated at the transistor is given by

$$P_{loss} = (V - u) \cdot i = \frac{1}{R} (V - u) \cdot u$$

Figure: Analogue power amplifier for driving the armature voltage





# Analog Amplifier (Why not)

$$P_{loss} = (V - u) \cdot i = \frac{1}{R} (V - u) \cdot u$$

- The Figure below plots the internal power loss at the transistor against the armature voltage.
- The power loss becomes the largest in the middle, where half the supply voltage  $V/2$  acts on the armature.
- This large heat loss is not only wasteful but also harmful, burning the transistor in the worst case scenario.
- Therefore, this type of linear power amplifier is seldom used except for driving very small motors.

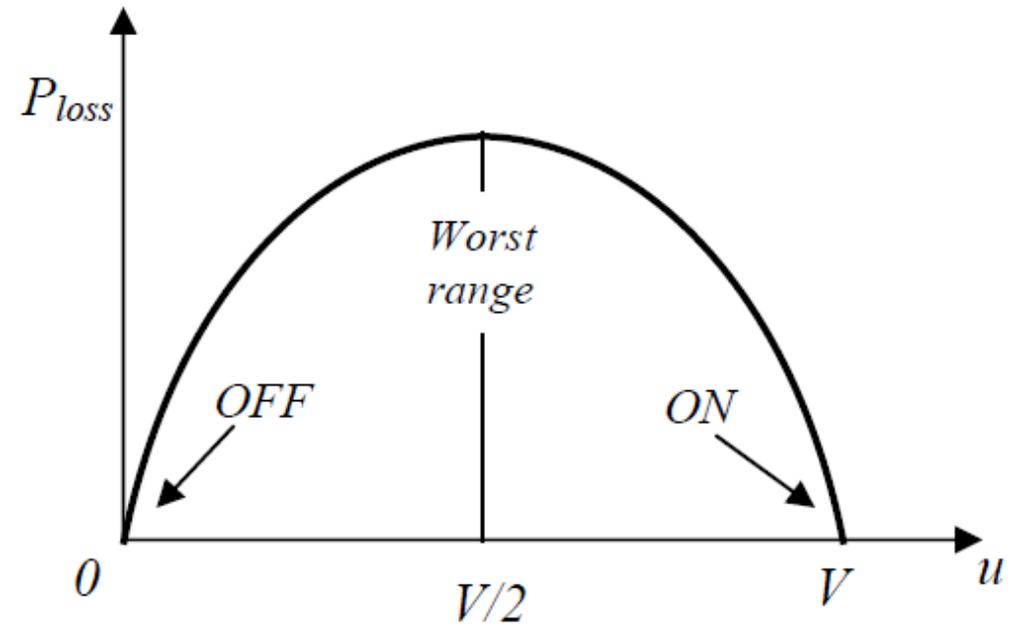


Figure: Power loss at the transistor vs. the armature voltage



# Pulse width modulation (PWM)

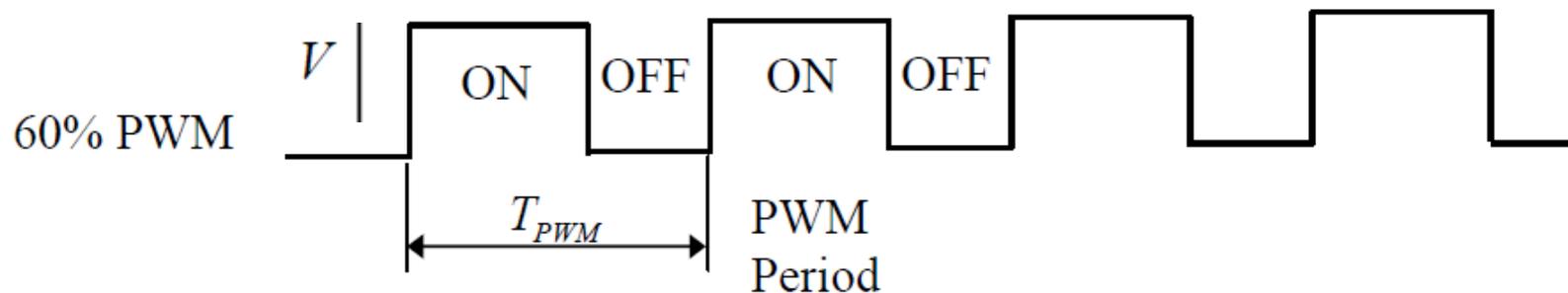
An alternative is to control the voltage via ON-OFF switching.

Pulse Width Modulation, or PWM for short, is the most commonly used method for varying the average voltage to the motor.

In the previous configuration(analog amplifier) it is clear that the heat loss is zero when the armature voltage is either  $0$  or  $V$ . This means that the transistor is completely shutting down the current (OFF) or completely admitting the current (ON).

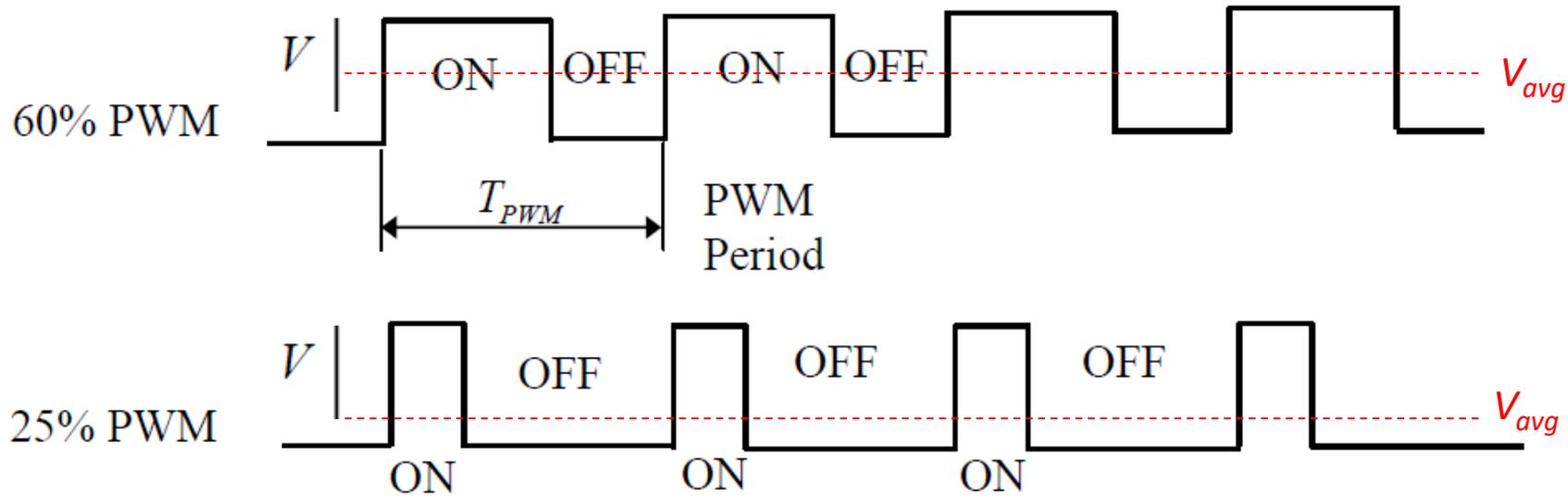
**Pulse Width Modulation (PWM )** is a technique to control an effective armature voltage by using the ON-OFF switching alone.

It varies the ratio of time length of the complete ON state to the complete OFF state. See Figure below





# Pulse width modulation (PWM)



A single cycle of ON and OFF states is called the PWM period, whereas the percentage of the ON state in a single period is called *duty rate*.

The first PWM signal is of 60% duty, and the second one is 25%. If the supply voltage is  $V=10$  volts, the average voltage is 6 volts and 2.5 volts, respectively.



# Pulse width modulation (PWM)

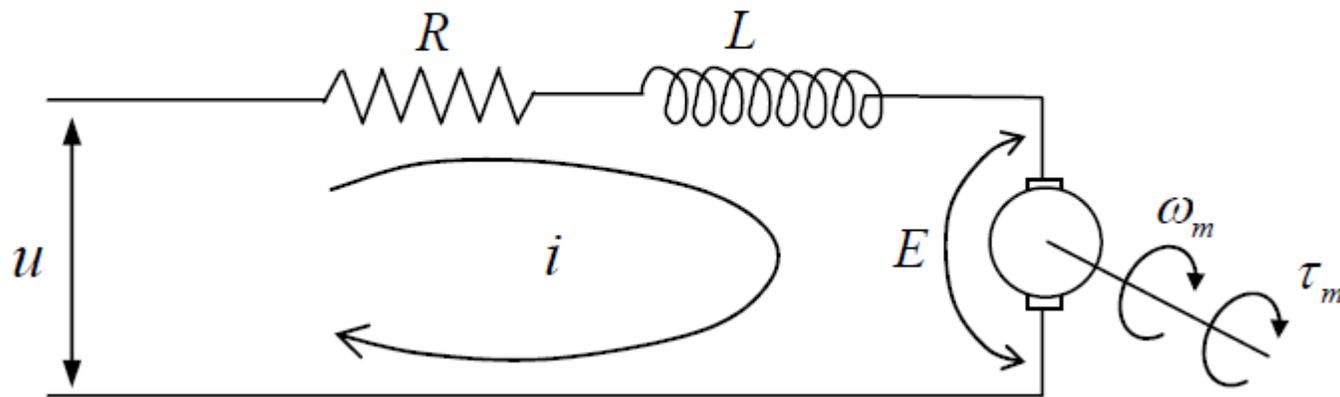
- The PWM period is set to be much shorter than the **time constant** associated with the mechanical motion.
- The PWM frequency, that is the reciprocal to the PWM period, is usually 2 ~ 20 kHz, whereas the bandwidth of a motion control system is at most 100 Hz.
- Therefore, the discrete switching does not influence the mechanical motion in most cases.

$$u = R \cdot i + L \frac{di}{dt} + E$$

$$\frac{K_t}{R} u = \tau_m + T_e \frac{d\tau_m}{dt} + \frac{K_t^2}{R} \omega_m$$

$$T_e = \frac{L}{R}$$

**time constant or reactance**



Electric circuit of armature



# Pulse width modulation (PWM)

-If the electric time constant  $T_e$  is much larger than the PWM period, the actual current flowing to the motor armature is a smooth curve, as illustrated in Figure (a).

In other words, the inductance works as a low-pass filter, filtering out the sharp ON-OFF profile of the input voltage.

-In contrast, if the electric time constant is too small, compared to the PWM period, the current profile becomes zigzag, following the rectangular voltage profile, as shown in Figure (b). As a result, unwanted high frequency vibrations are generated at the motor rotor.

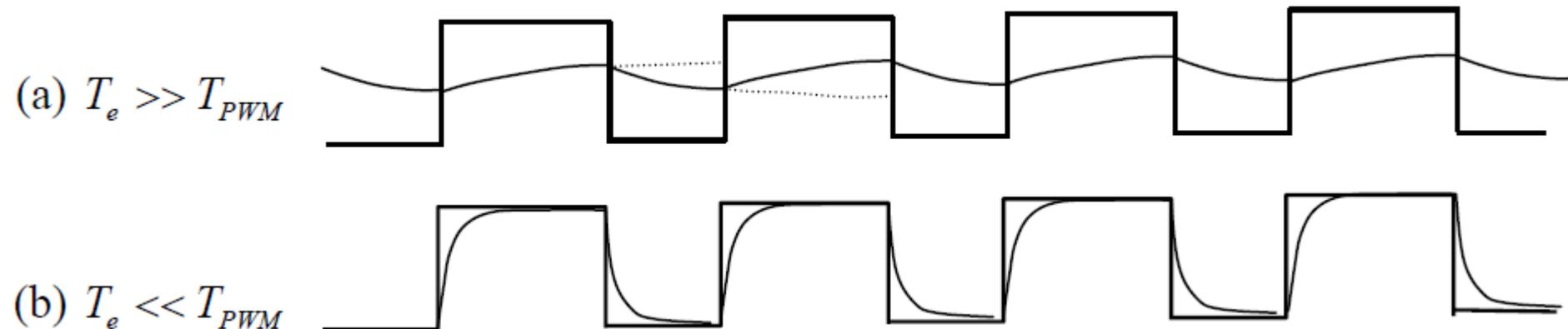


Figure: Current to the motor is smoothed due to the armature inductance  $L$ .



# PWM switching characteristics

As the PWM frequency increases, the current driven to the motor becomes smoother, and the nonlinearity due to discrete switching disappears.

Furthermore, high PWM frequencies cause no audible noise of switching. The noise disappears as the switching frequency becomes higher than the human audible range, say 15 kHz. Therefore, a higher PWM frequency is in general desirable.

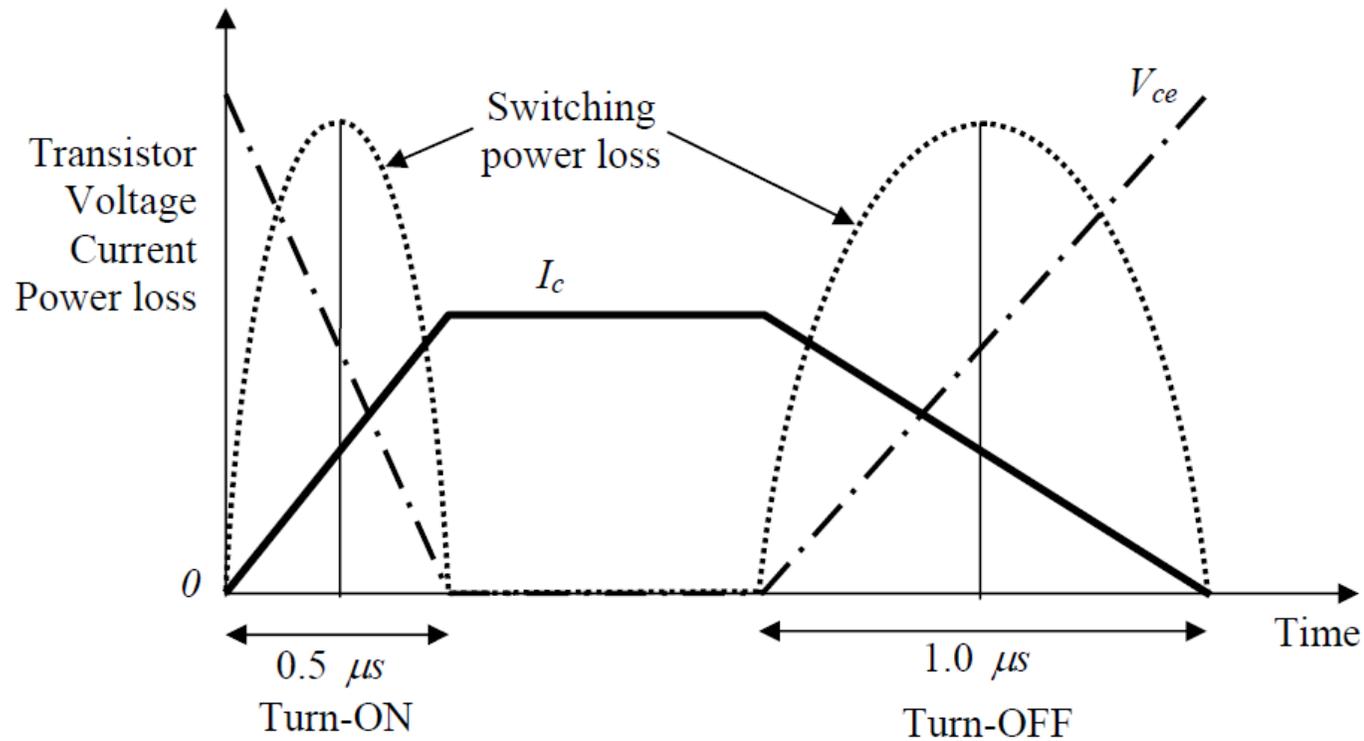
However, it causes a few adverse effects. As the PWM frequency increases:

- The **heat loss** increases and the transistor may over-heat,
- Harmful **large voltage spikes** and noise are generated, and
- **Radio frequency interference** and electromagnetic interference become prominent.



# PWM switching characteristics

The first adverse effect is the most critical one, which limits the capacity of a PWM amplifier. Although no power loss occurs at the switching transistor when it is completely ON or OFF, a significant amount of loss is caused during transition.



Transient responses of transistor current and voltage and associated power loss during turn-on and turn-off state transitions



# PWM switching characteristics

- It is clear that a switching transistor having **fast turn-on and turn-off characteristics is desirable**, since it causes less power loss and heat generation.
- Power *MOSFETs* (*Metal-Oxide-Semiconductor Field-Effect Transistors*) have very fast switching characteristics, enabling 15 ~ 100 kHz of switching frequencies. For relatively small motors, MOSFETs are widely used in industry due to their fast switching characteristics.
- For larger motors, **IGBTs (Insulated Gate Bipolar Transistor)** are the rational choice because of their larger capacity and relatively fast response.

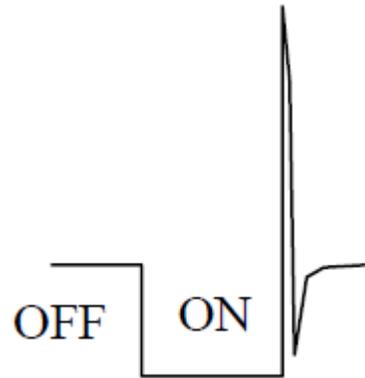


# PWM switching characteristics

As the switching speed increases, the heat loss becomes smaller. However, fast switching causes other problems. Consider the dynamic equation of the armature:

$$u = R \cdot i + L \frac{di}{dt} + E$$

High speed switching means that the time derivative of current  $i$  is large. This generates a large inductance-induced kickback voltage  $L \frac{di}{dt}$  that often damages switching semiconductors.

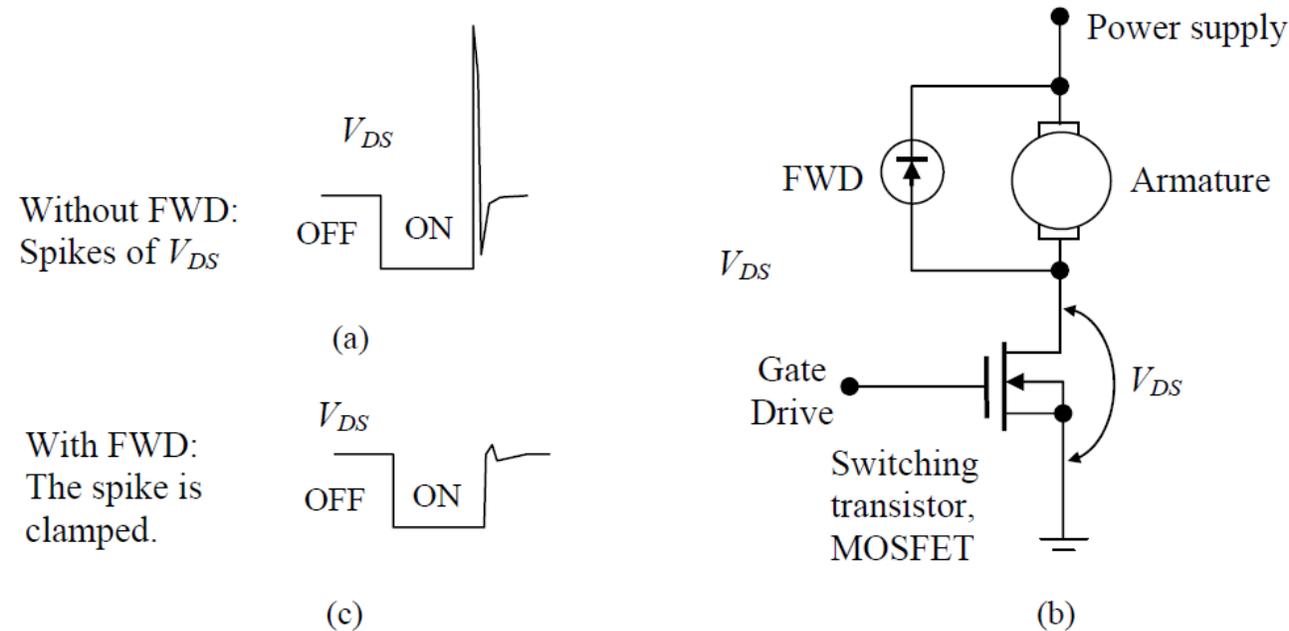




# PWM switching characteristics

As illustrated in Figure (a), a large spike is induced when turning on the semiconductor.

To get rid of this problem a **free-wheeling-diode (FWD)** is inserted across the motor armature, as shown in Figure (b). As the voltage across the armature exceeds a threshold level, FWD kicks in to bypass the current so that the voltage may be clamped, as shown in figure (c).



Voltage spike induced by inductance (a), free-wheeling diode (b), and the clamped spike with FWD (c)



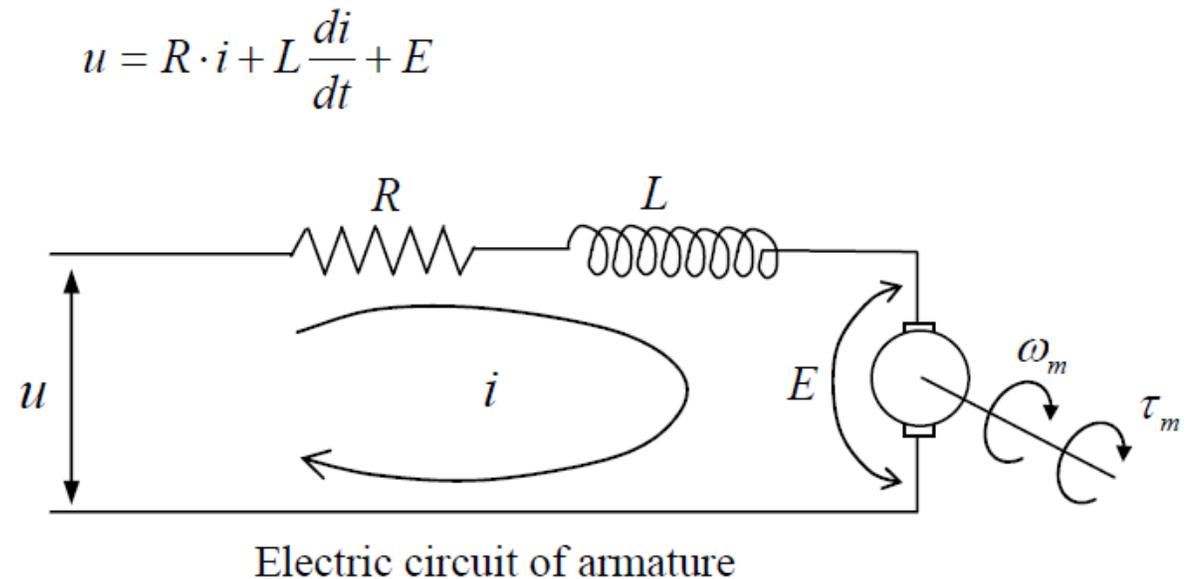
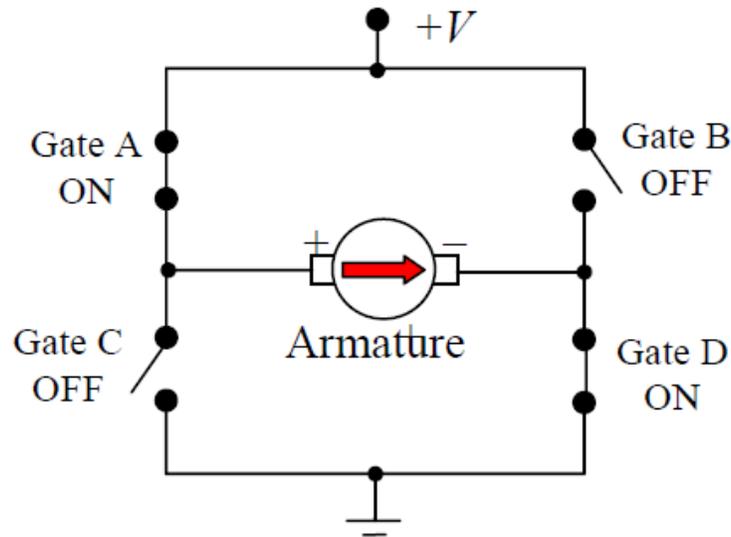
# The H-bridge and bipolar PWM amplifiers

In most robotics applications, **bi-directional control** of motor speed is necessary.

This requires a PWM amplifier to be bipolar, allowing for both forward and backward rotations.

The architecture described in the previous section needs to be extended to meet this bipolar requirement. The H-Bridge architecture is commonly used for bipolar PWM amplifiers.

As shown in Figure below, the **H-Bridge** architecture resembles the letter H in the arrangement of switching transistors around the motor armature.



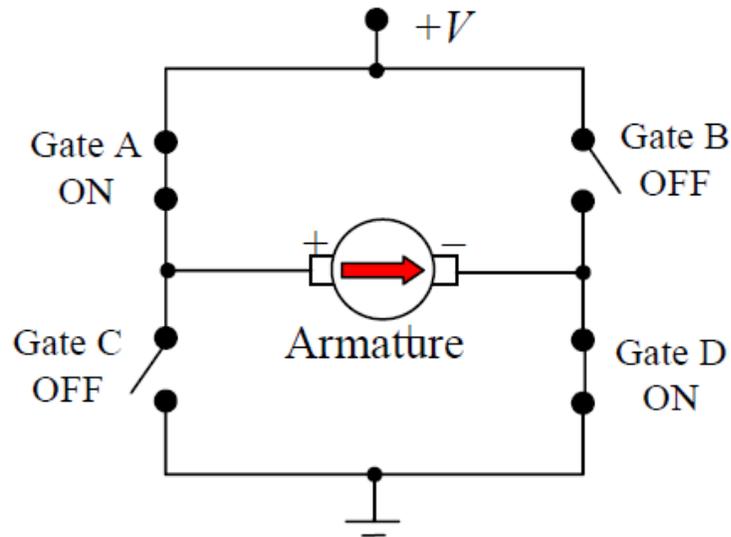


# The H-bridge and bipolar PWM amplifiers

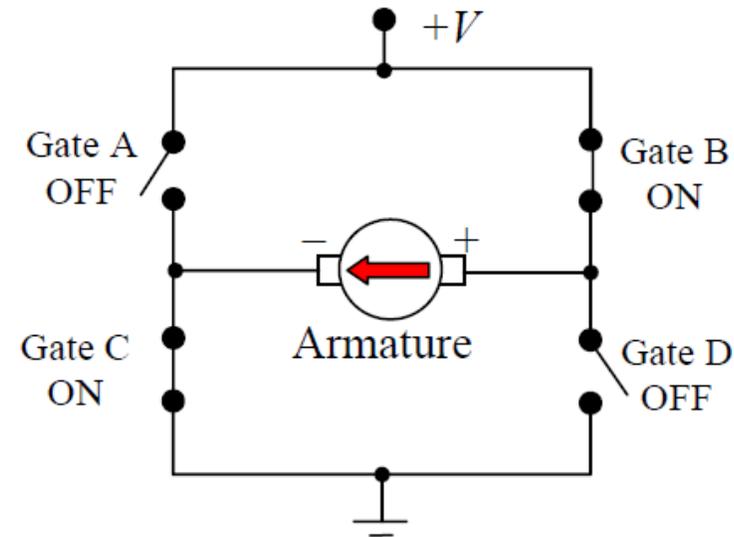
Switching transistors A and B are pulled up to the supply voltage  $V$ , whereas transistors C and D are connected to ground. Combinations of these four switching transistors provide a variety of operations.

In figure (i), gates A and D are ON, and B and C are OFF. This gate combination delivers a current to the armature in the **forward** direction.

When the gate states are reversed, as shown in figure (ii), the direction of current is reversed and the motor moves **backward**.



(i) Forward motion



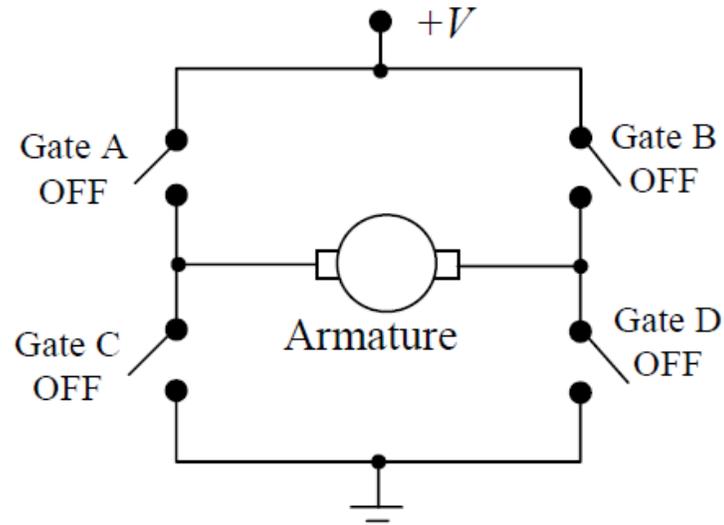
(ii) Reverse motion



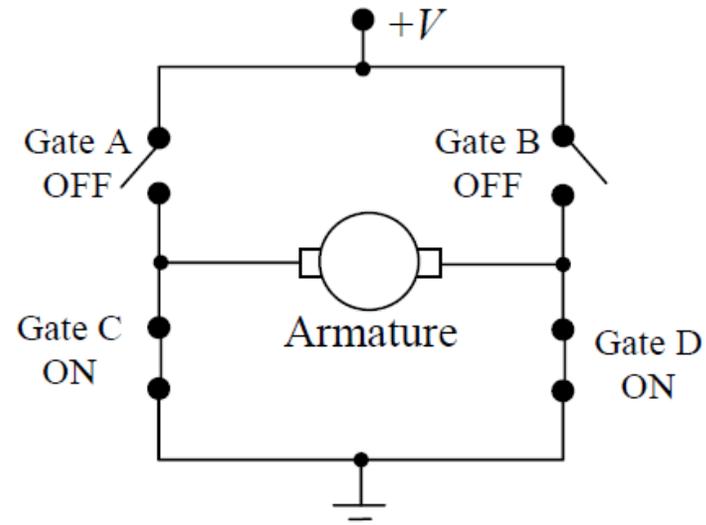
# The H-bridge and bipolar PWM amplifiers

Furthermore, the motor coasts off when all the gates are turned OFF, since the armature is totally isolated or disconnected as shown in figure (iii).

On the other hand, the armature windings are shortened, when both gates C and D are turned ON and A and B are turned OFF. See figure (iv). This shortened circuit provides a “**braking**” effect, when the motor rotor is rotating.



(iii) The motor armature coasts off



(iv) The motor windings are shortened causing a braking effect.



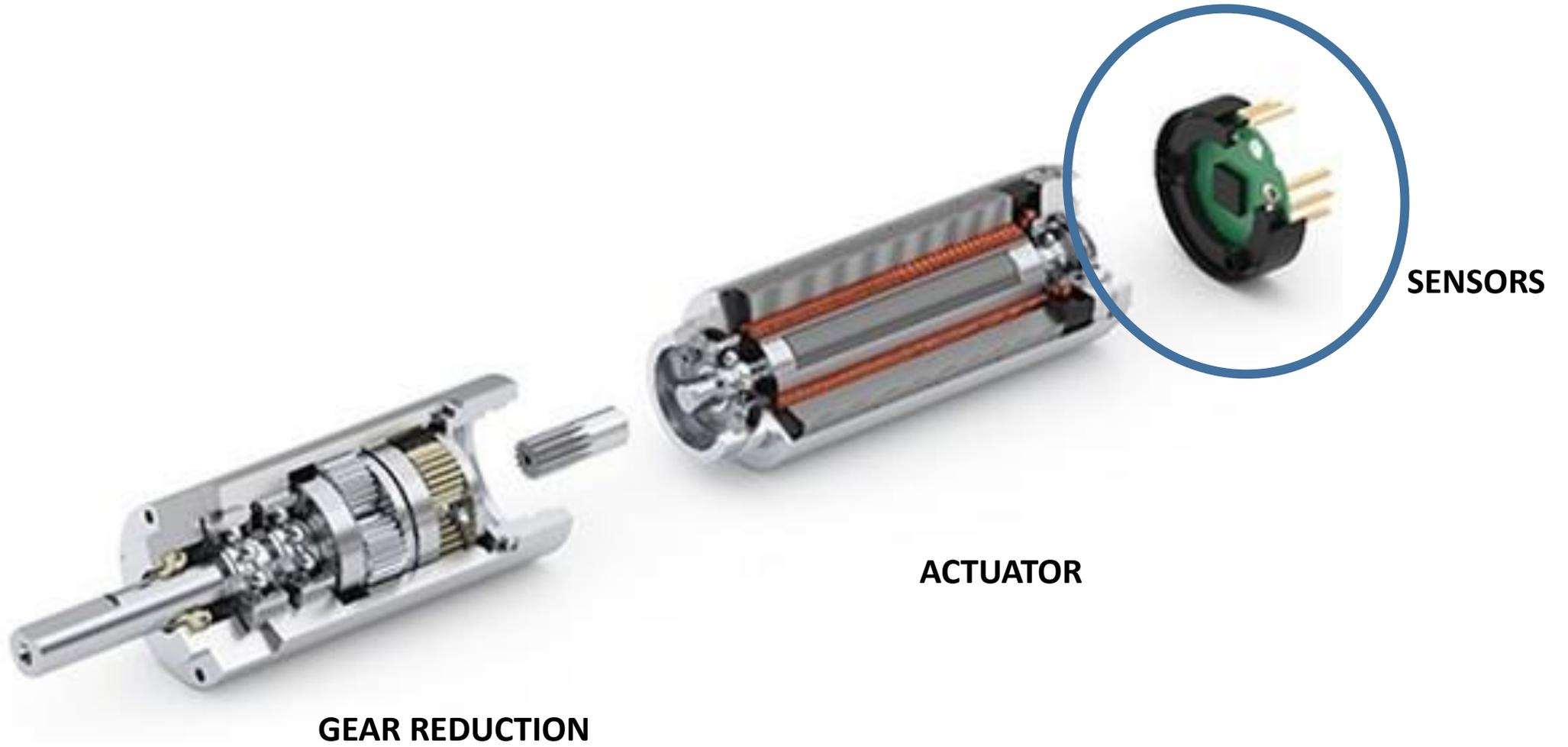
# Instructional videos on PWM

<https://www.youtube.com/watch?v=GQLED3gmONg>

<https://www.youtube.com/watch?v=YmPziPfaByw>



# DC Motor: sensors





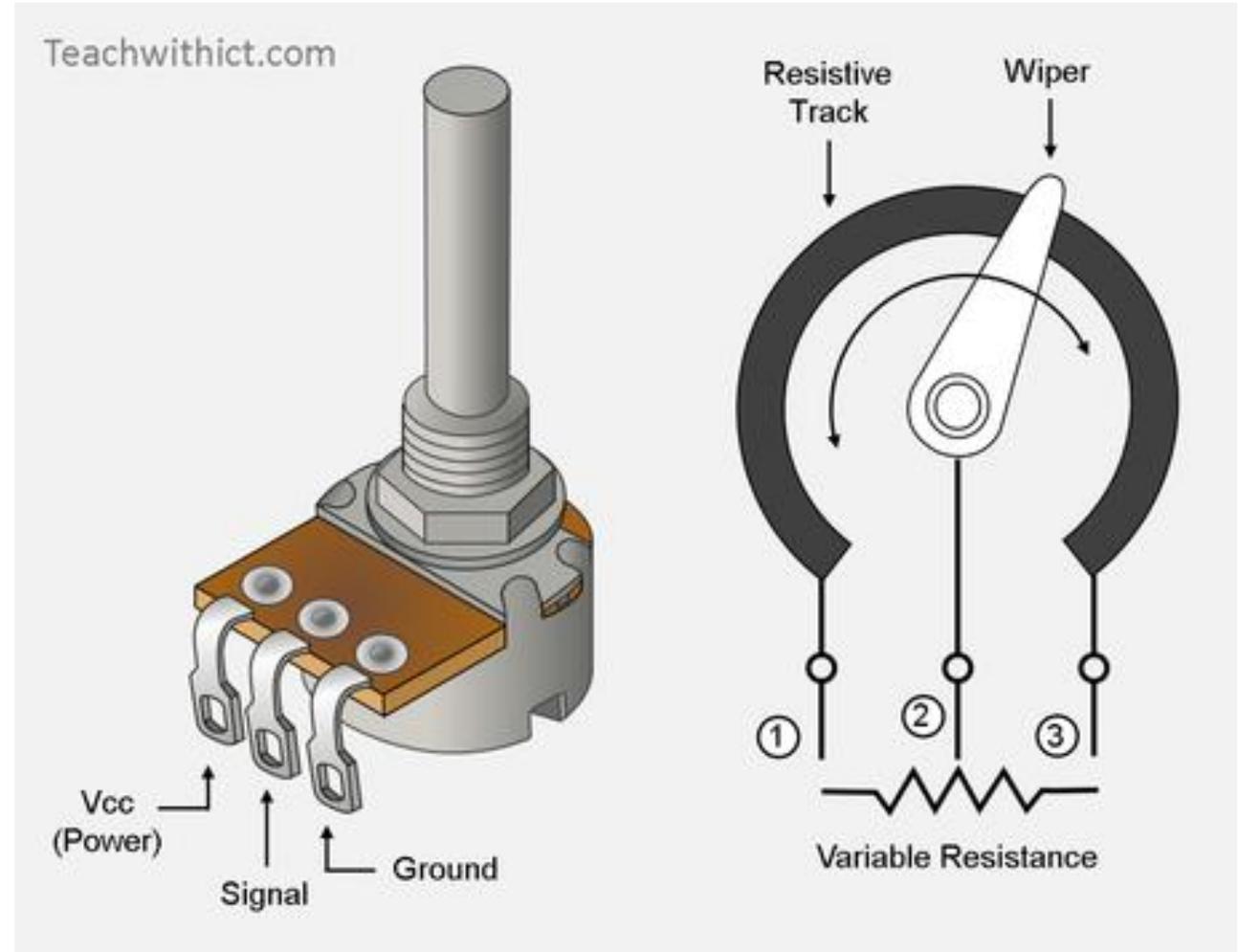
# Once upon a time...potentiometers

The initial servomechanism were based on analogue feedback technology, using a potentiometer

These analogue feedbacks, are no longer used in industrial robots and other industrial applications, due to limited reliability and performance.

A potentiometer, is poor in reliability, resolution, accuracy, and signal to noise ratio. The output tap of the variable resistance slides on a track of resistive material, making a mechanical contact all the time.

This slide contact causes not only electric noise but also wear of the contacting surfaces. The resolution and linearity depend on the uniformity of the resistive material coated on the substrate, and that is a limiting factor.



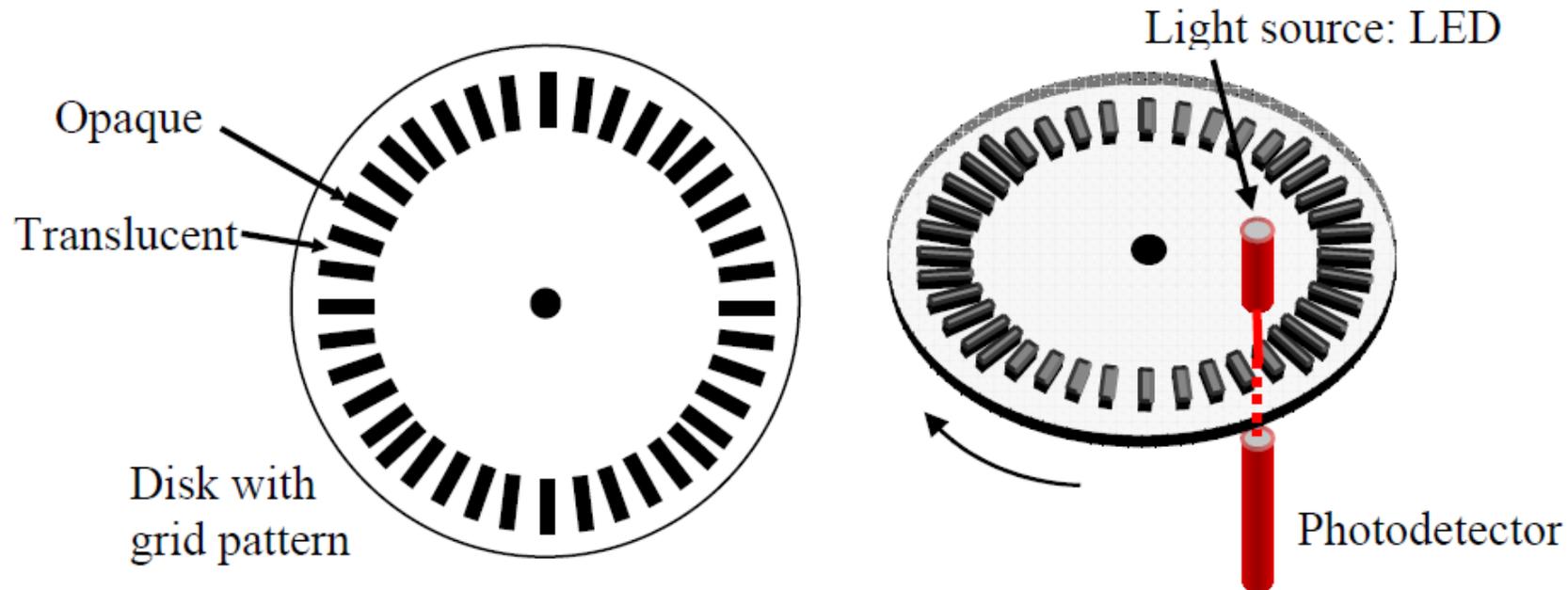


# Optical Shaft Encoders

Today's industrial standard is optical shaft encoders, having no sliding contact.

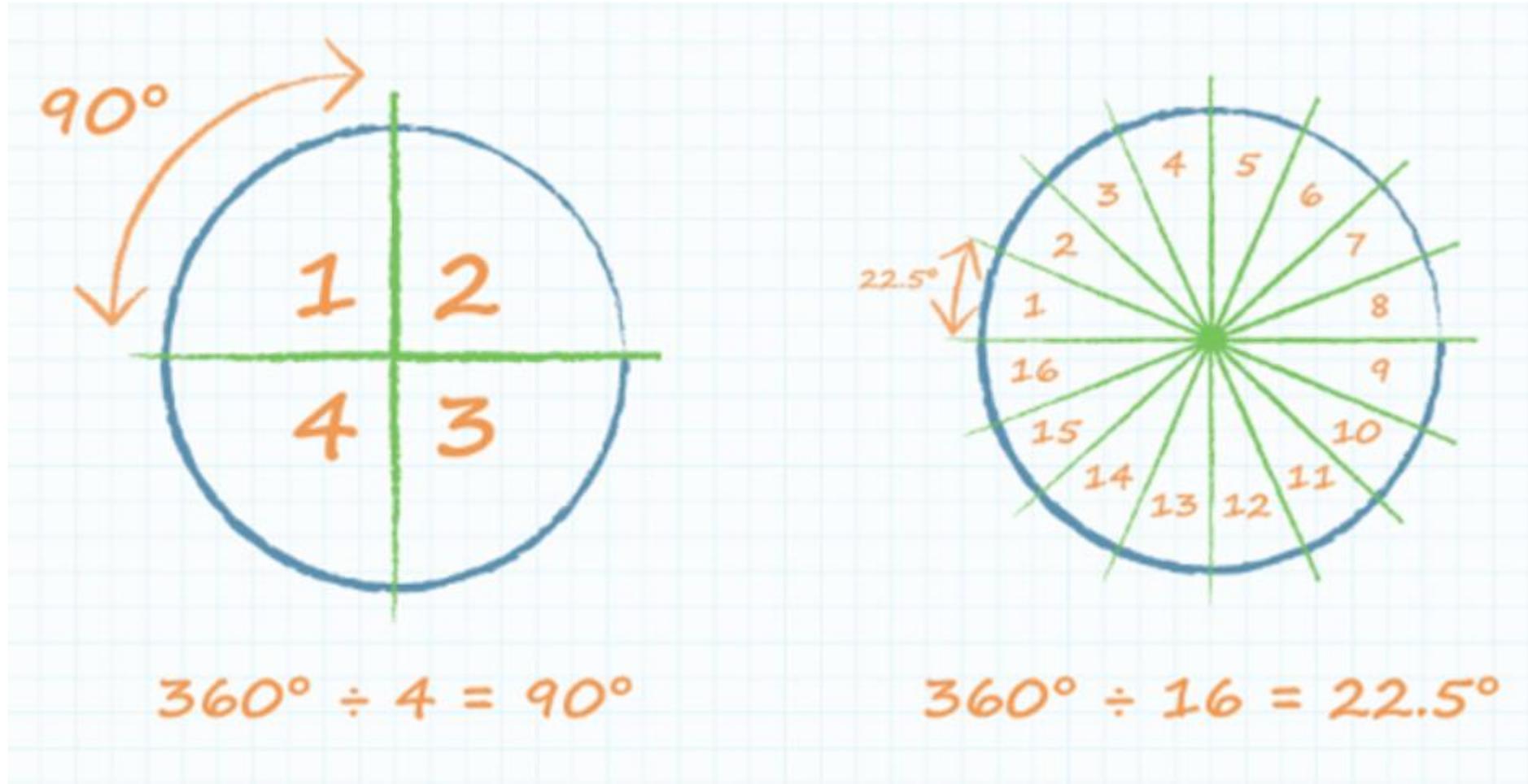
An **optical encoder** consists of a rotating disk with grids, light sources, photodetectors, and electronic circuits

- A pattern of alternating opaque and translucent grids is printed on the rotating disk.
- A pair of light source and photodetector is placed on both sides of the rotating disk.
- As an opaque grid comes in, the light beam is blocked, while it is transmitted through the disk, when the translucent part comes in. The light beam is then detected by the photodetector.





# Optical Shaft Encoders: resolution

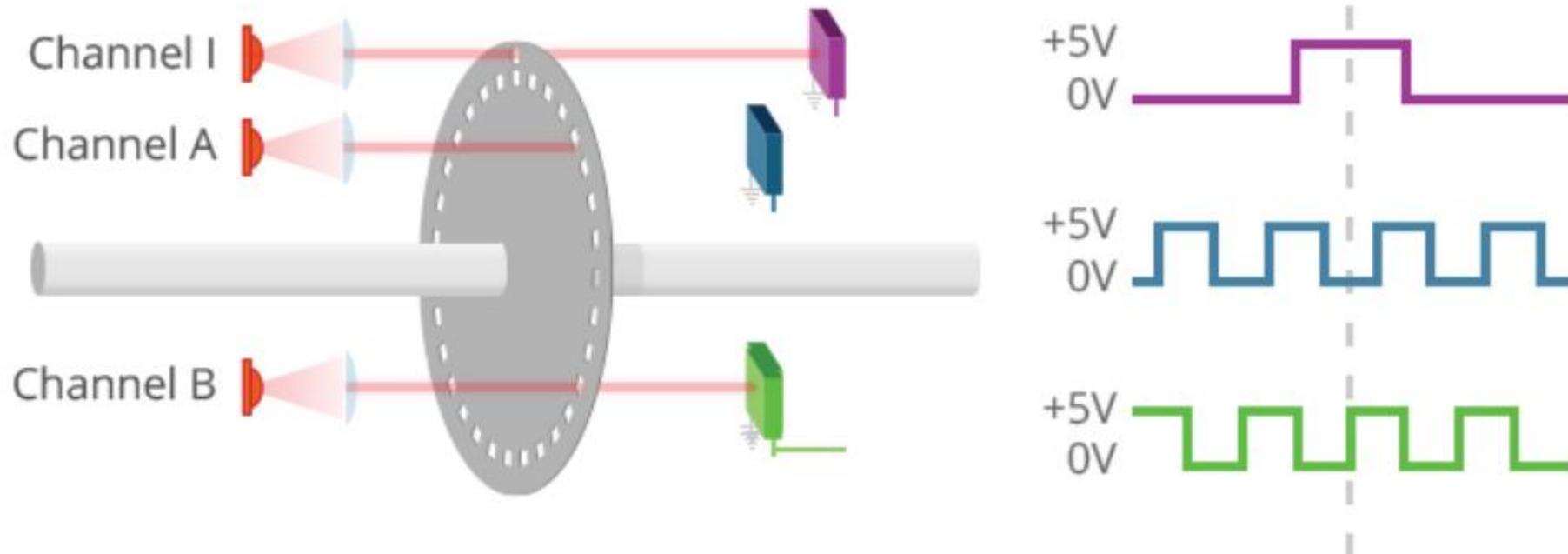




# Optical Shaft Encoders

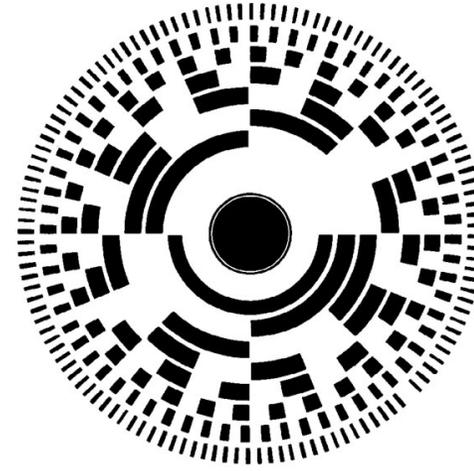
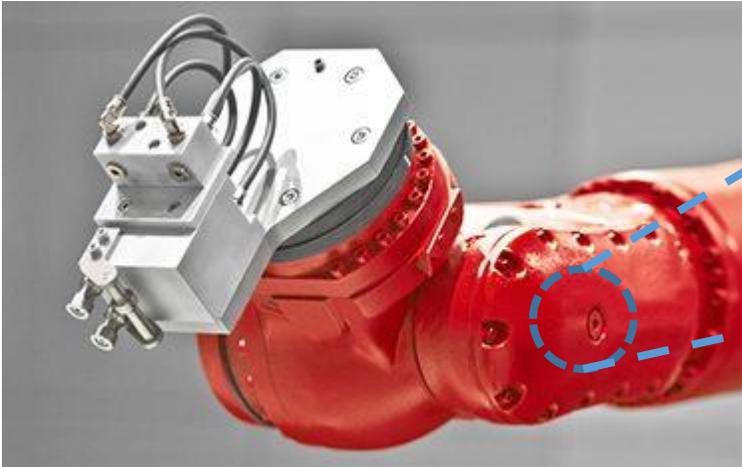
An encoder is a device that senses mechanical motion. It translates mechanical motion such as position, speed, distance, and direction into electrical signals.

Inside a rotary encoder there is a disc fixed to a shaft that is free to rotate. On one side of the disc is a signal source, on the other side a receiver. As the disc turns, the signal source is alternately allowed to pass and be blocked. When the signal is passed through the disc, an output pulse is generated.





# Optical Shaft Encoders



The disk is coupled to a motor shaft or a robot joint to measure. As it rotates, an **alternating ON-OFF** signal is obtained with the photodetector.

The number of grids passing through the optical elements represents the distance travelled.

- This optical shaft encoder has no mechanical component making a slide contact, and has no component wear.
- An optical circuit is not disturbed by electric noise, and the photodetector output is a digital signal, which is more stable than an analogue signal.
- These make an optical shaft encoder reliable and robust; it is a suitable choice as a feedback sensor for servomotors.



# Optical Shaft Encoders: Position measurement

- One problem with the above optical encoder design is that the **direction of rotation cannot be distinguished from the single photodetector output.**
- The **photodetector output is the same for both clockwise and counter-clockwise rotations.**
- **There is no indication as to which way the disk is rotating.** Counting the pulse number merely gives the total distance the shaft has rotated back and forth.
- 
- To measure the angular “position”, the direction of rotation **must be distinguished.**

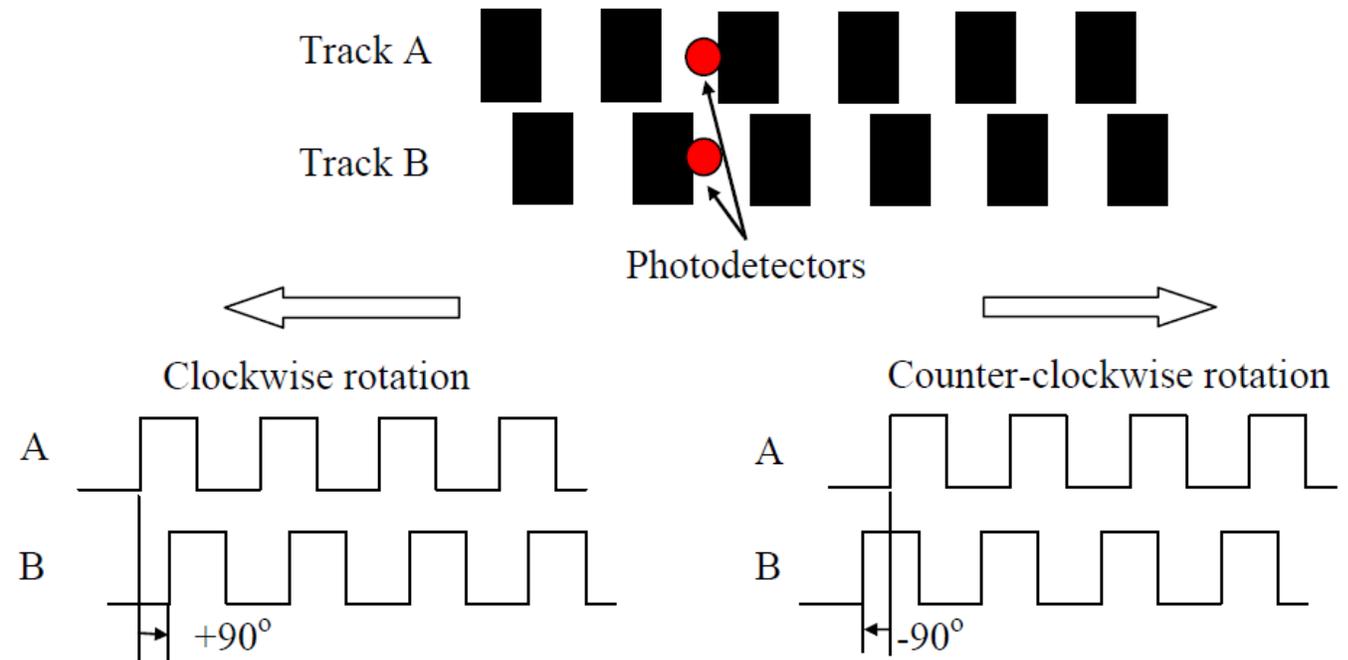
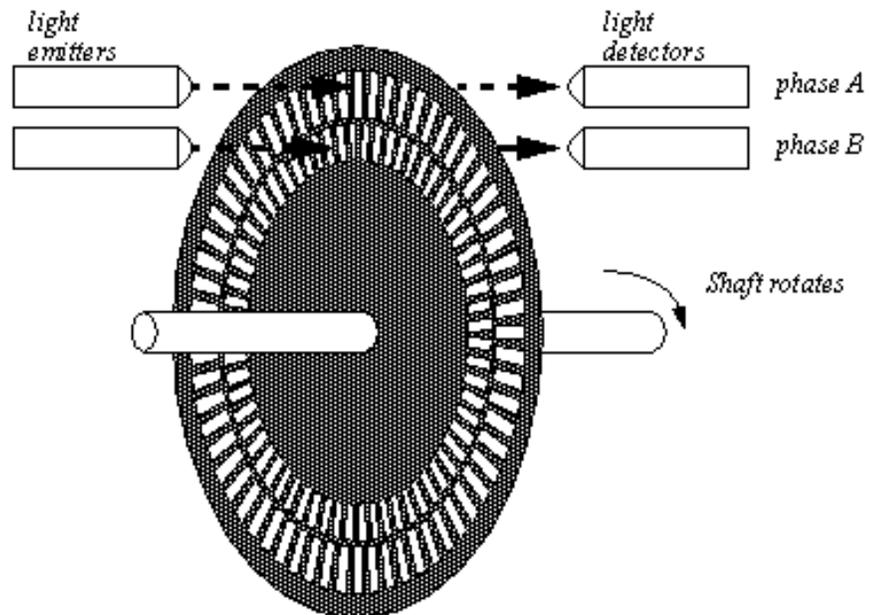


# Optical Shaft Encoders: Position measurement

Solution is to add another pair of light source/photodetector and a second track of opaque/translucent grids with 90 degrees of phase difference from the first track.

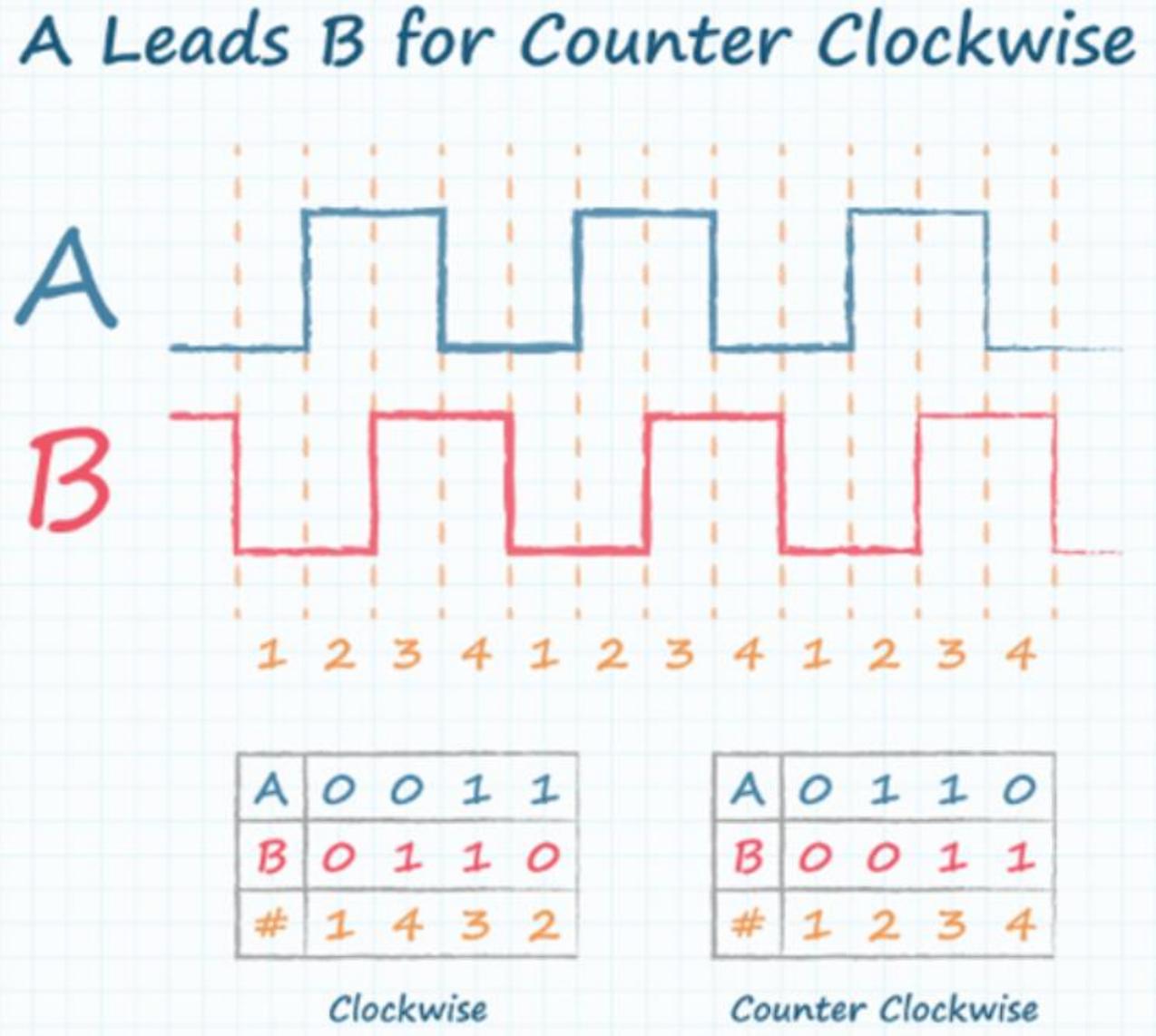
The Figure illustrates a double track pattern and resultant output signals for clockwise and counter-clockwise rotations.

Note that track A leads track B by 90 degrees for clockwise rotation and that track B leads track A for counter-clockwise rotation. By detecting the phase angle the direction of rotation can be distinguished, and this can be done easily with an up-down counter.



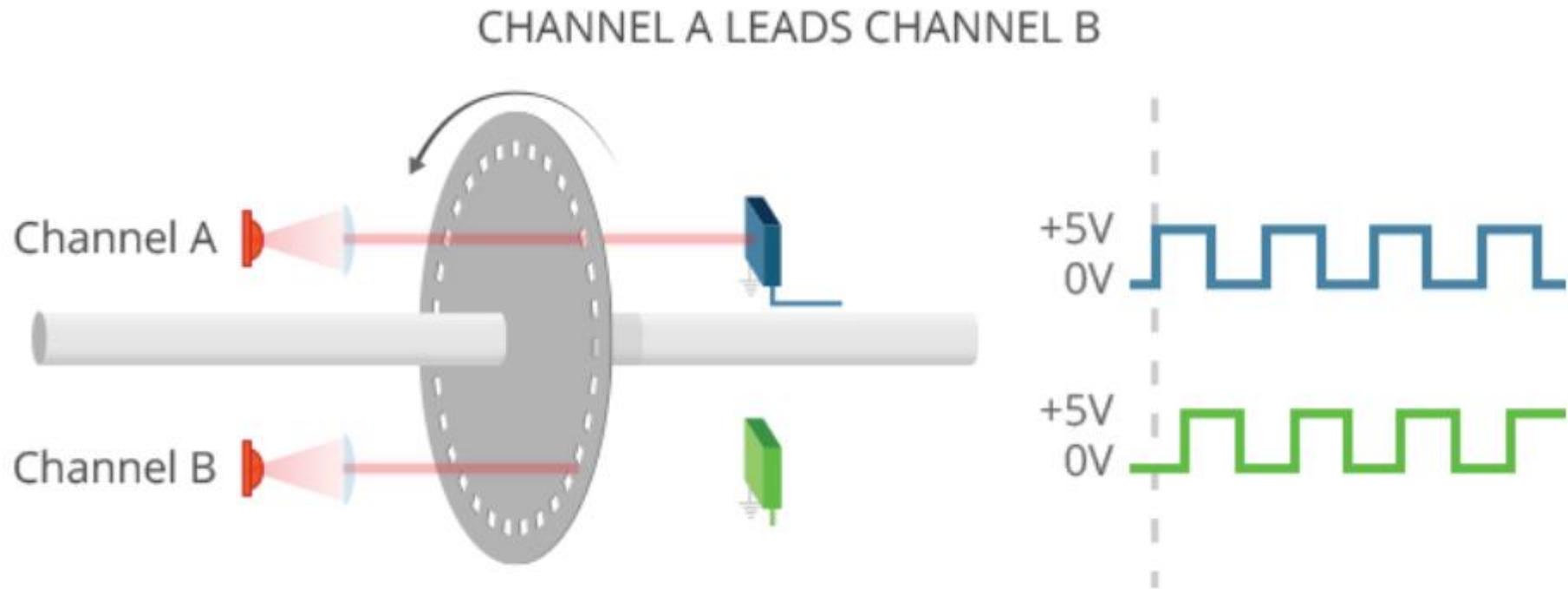


# Optical Shaft Encoders: Position measurement





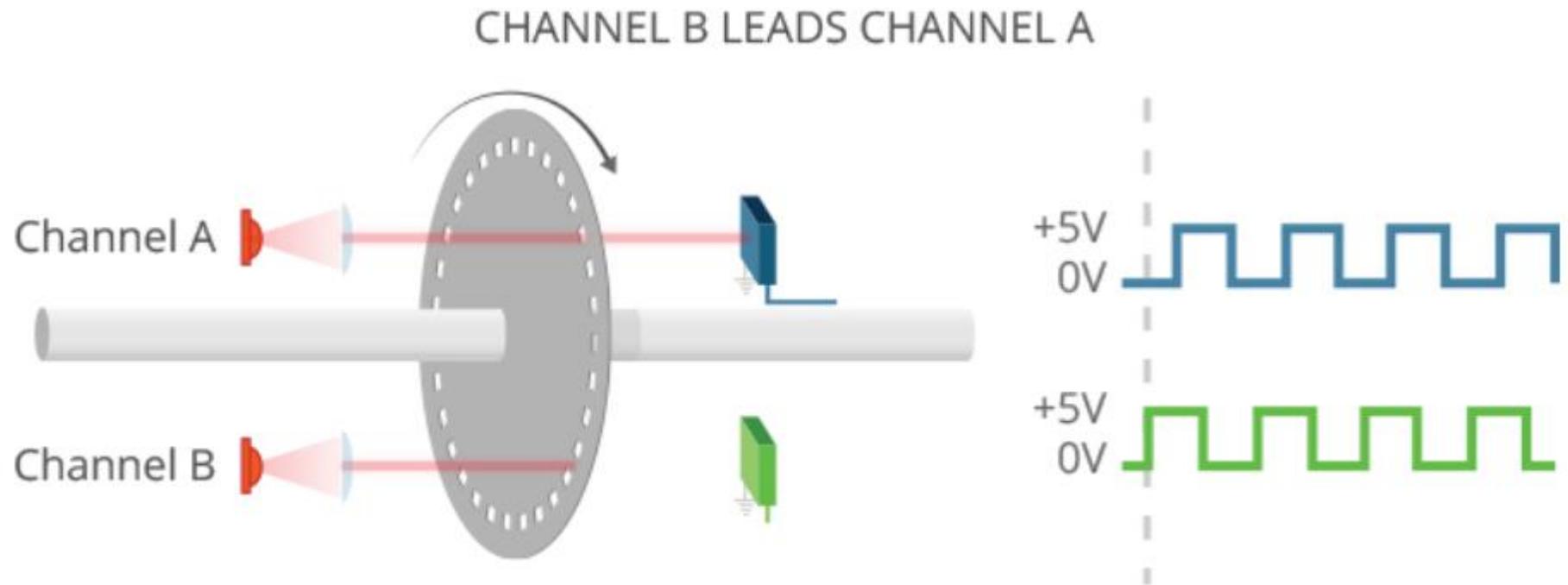
# Optical Shaft Encoders: rotation measurement



- In this example, Channel A leads B, i.e., Channel A outputs a signal before Channel B. This indicates the shaft is rotating counter-clockwise.



# Optical Shaft Encoders: rotation measurement



- In this example, Channel B leads A. This indicates the shaft is rotating clockwise.

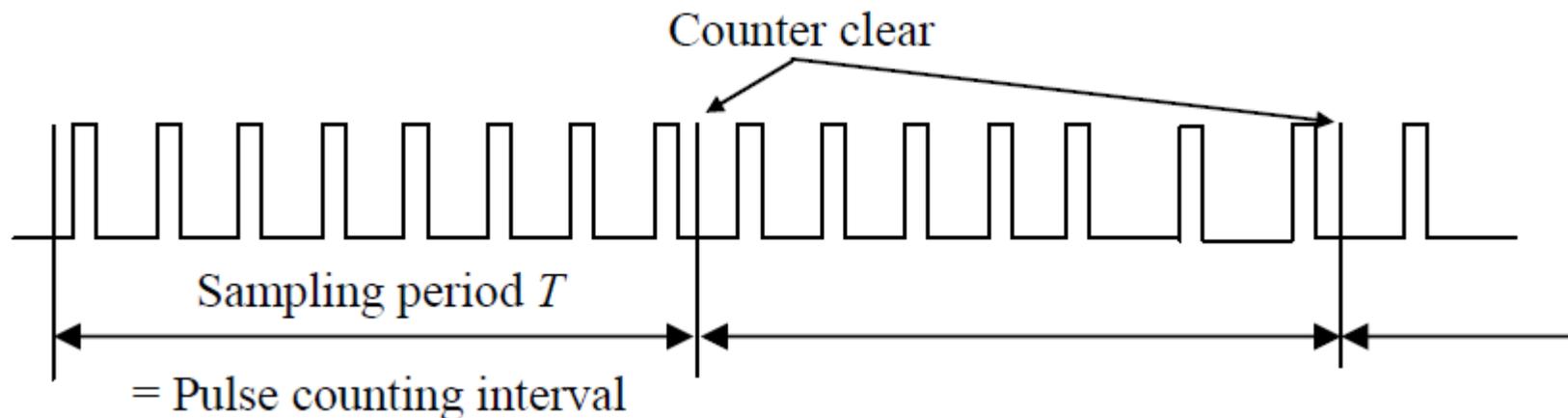


# Optical Shaft Encoders: Speed Information

Velocity feedback is needed for improving accuracy of speed control as well as for compensating for system dynamics.

A salient feature of optical encoders is that velocity information can be obtained along with position measurement.

The density of the pulse train, i.e. the pulse frequency, is approximately proportional to the angular velocity of the rotating shaft. The pulse density can be measured by counting the number of incoming pulses in every fixed period, say  $T=10\text{ ms}$ , as shown in the figure.

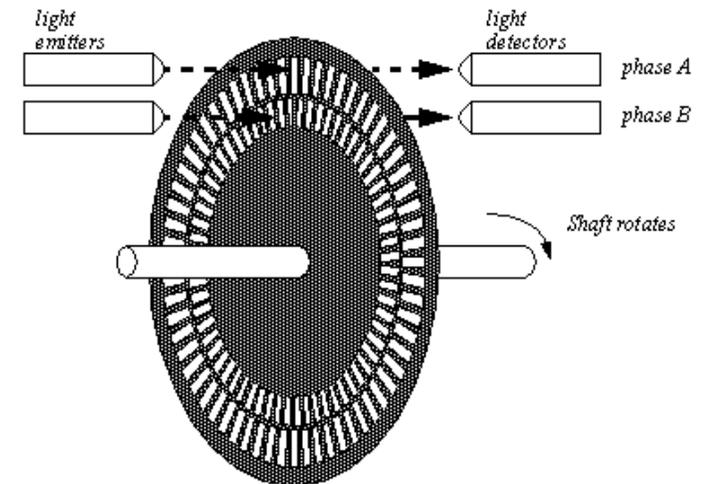




# Optical Shaft Encoders

By simply feeding both A phase and B phase encoder signals to an up-down counter, the **direction of rotation is first detected**, and the number of rising edges and falling edges of both signals is counted in such a way that the counter adds the incoming edge number for clockwise rotation and subtract the edge numbers for counter-clockwise rotation.

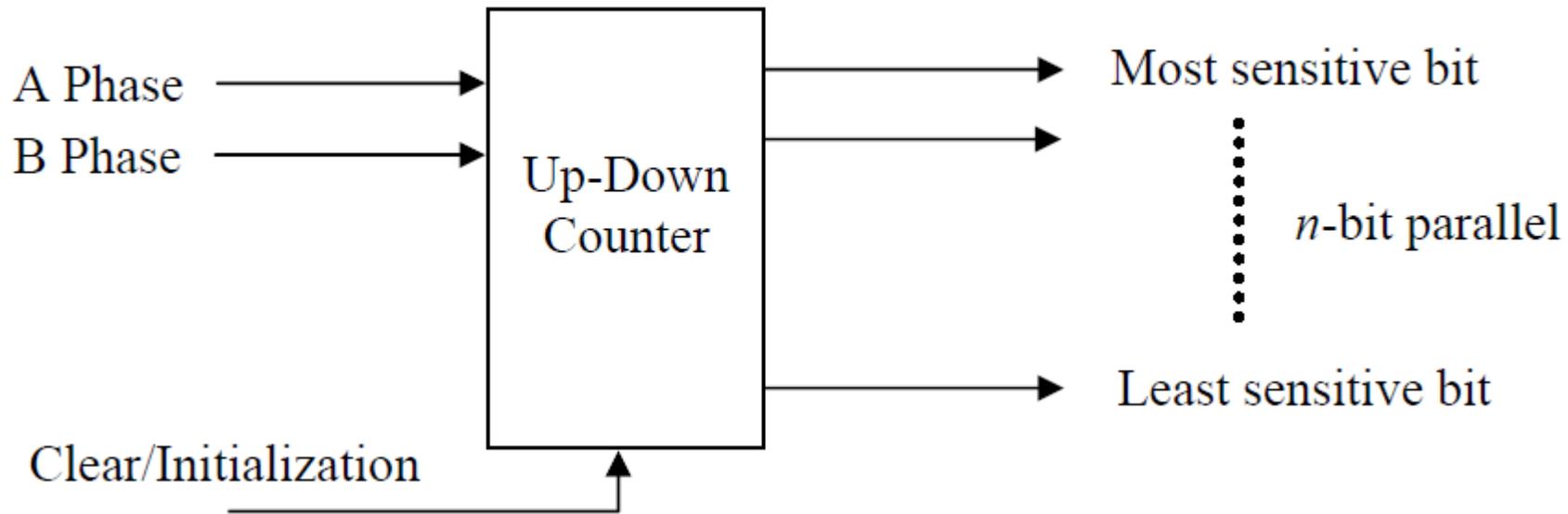
The **up-down counter indicates the cumulative number of edges**, that is, the angular “position” of the motor. The output of the up-down counter is binary  $n$ -bit signals ready to be sent to a digital controller without A/D conversion





# Optical Shaft Encoders (incremental)

- This type of encoder requires initialization of the counter prior to actual measurement.
- Usually a robot is brought to a home position and the up-down counters are set to the initial state corresponding to the home position.
- This type of encoder is referred to as an *incremental encoder*, since A-phase and B-phase signals provide relative displacements from an initial point.
- Whenever the power supply is shut down, the initialization must be performed for incremental encoders.

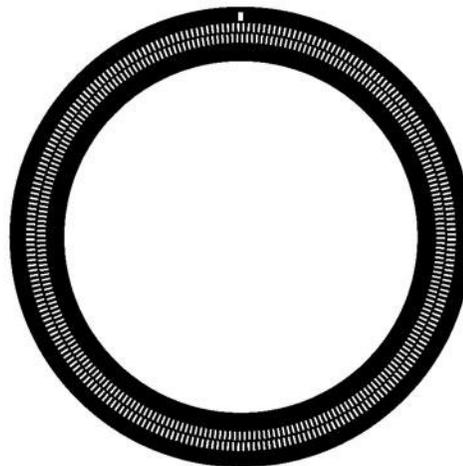




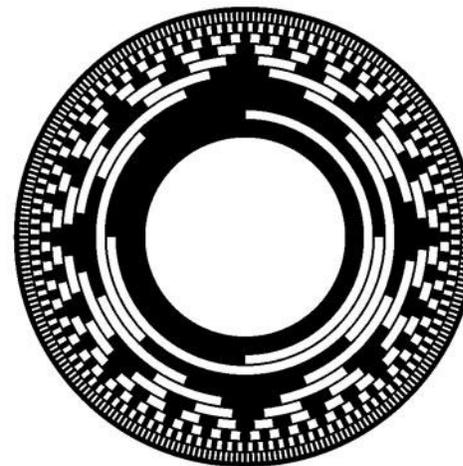
# Absolute encoders

An **absolute encoder** provides an  $n$ -bit absolute position as well as the direction of rotation without use of an up-down counter and initialization.

An **absolute encoder** provides an  $n$ -bit absolute position as well as the direction of rotation without use of an up-down counter and initialization. As shown in Figure the rotating disk has  $n$ -tracks of opaque-translucent grid patterns and  $n$  pairs of light sources and photodetectors. The  $n$ -tracks of grid patterns differ in the number of grids; the innermost track has only  $1=2^0$  pair of opaque and translucent slits, the second track has  $2=2^1$  pairs, and the  $i$ -th track has  $2^{i-1}$  pairs. The  $n$  outputs from the photodetectors directly indicate the  $n$ -bit absolute position of the rotating disk. In general, absolute encoders are more complex and expensive than incremental encoders. In case of power failure, incremental encoders need a laborious initialization procedure for recovery. For quick recovery as well as for safety, absolute encoders are often needed in industrial applications.



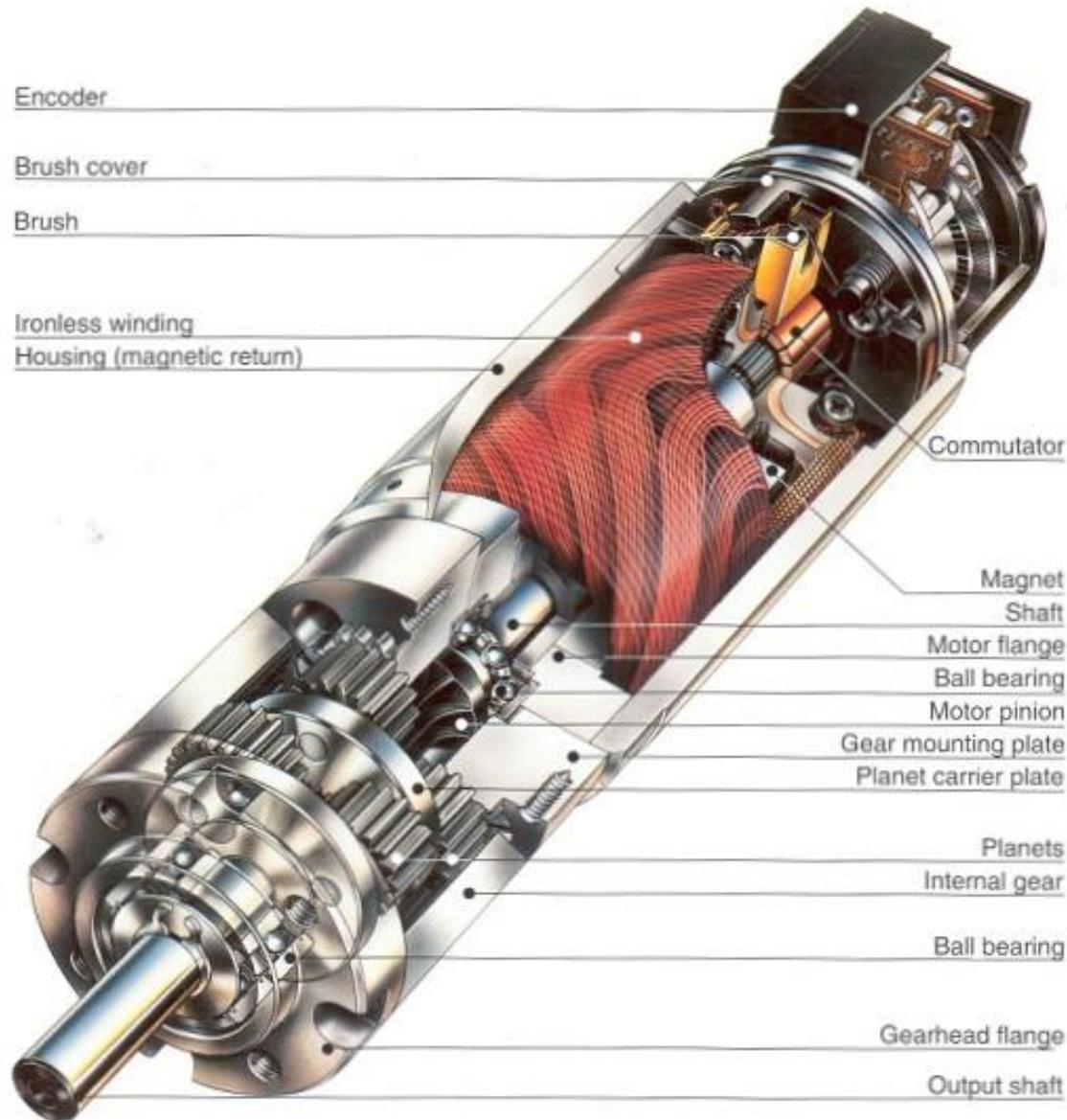
1 Incremental



2 Absolute



# Limitation of DC Motors



The DC motor described in the previous section is the simplest, yet efficient motor among various actuators applied to robotic systems.

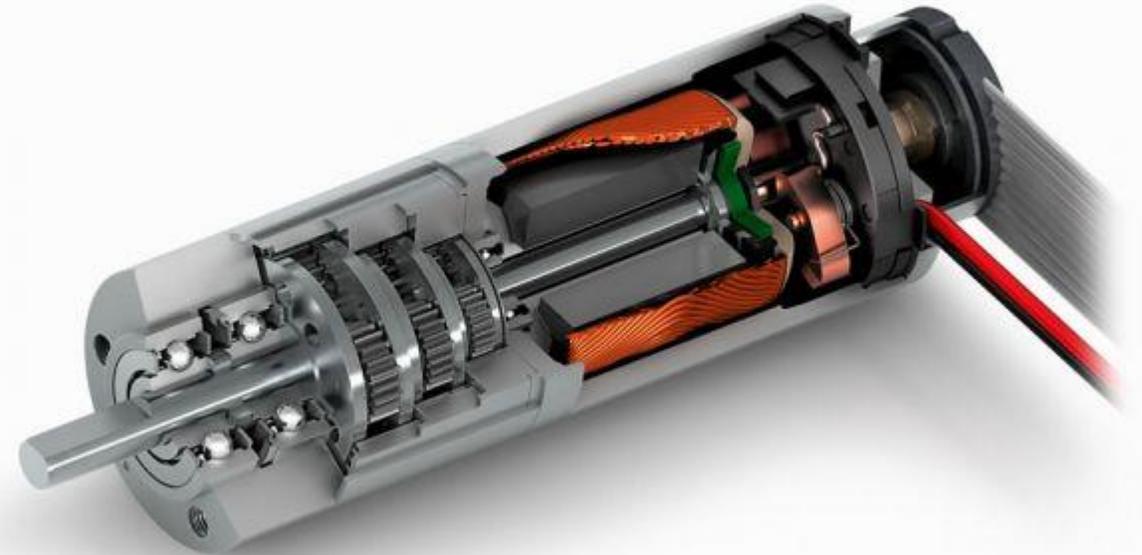
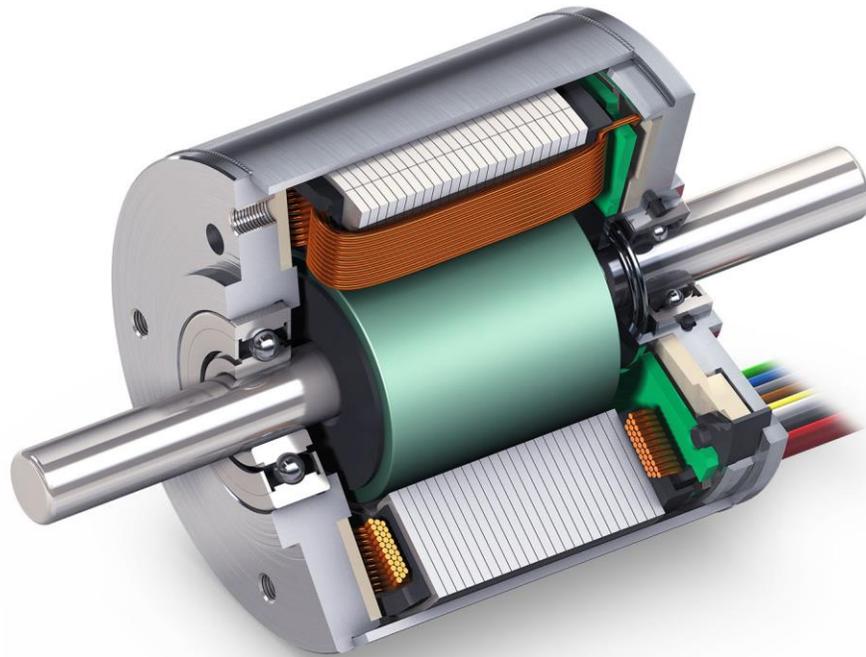
Traditional DC motors, however, are limited in reliability and robustness due to wear of the brush and commutation mechanism.

Also brushes introduce mechanical noise and friction limiting the efficiency of the actuation.



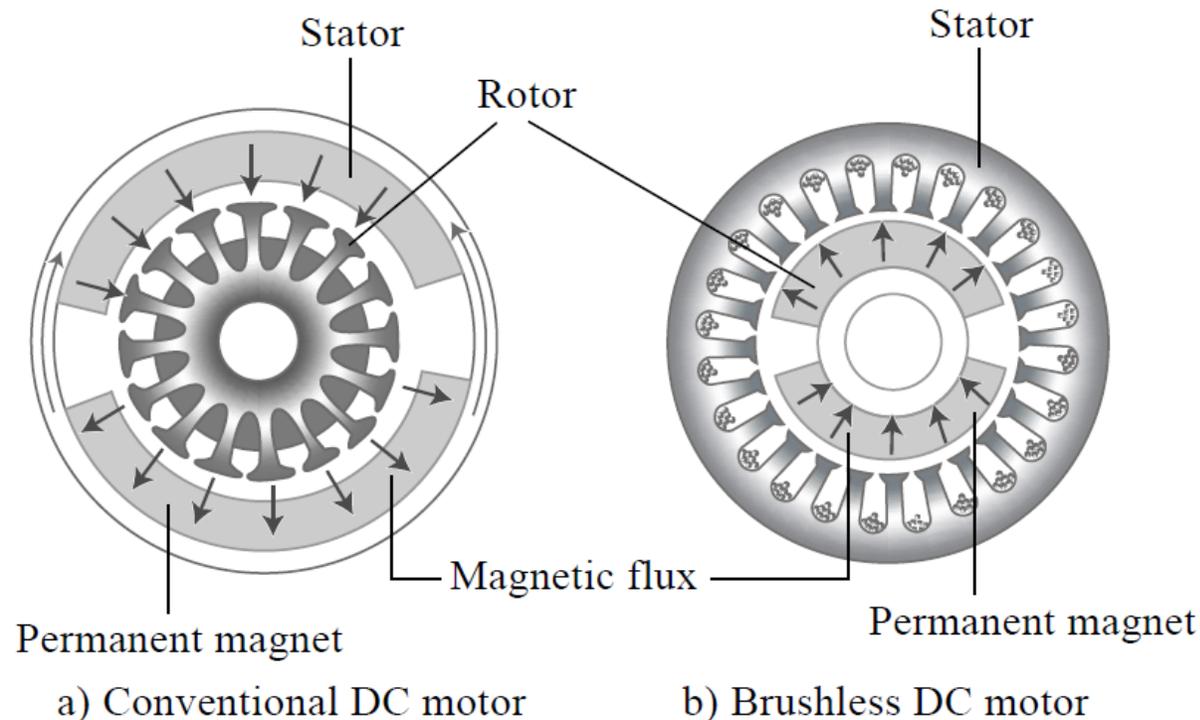
# Brushless DC Motors

In industrial applications where a high level of reliability and robustness is required, DC motors have been replaced by **brushless motors** and other types of motors having no mechanical commutator.



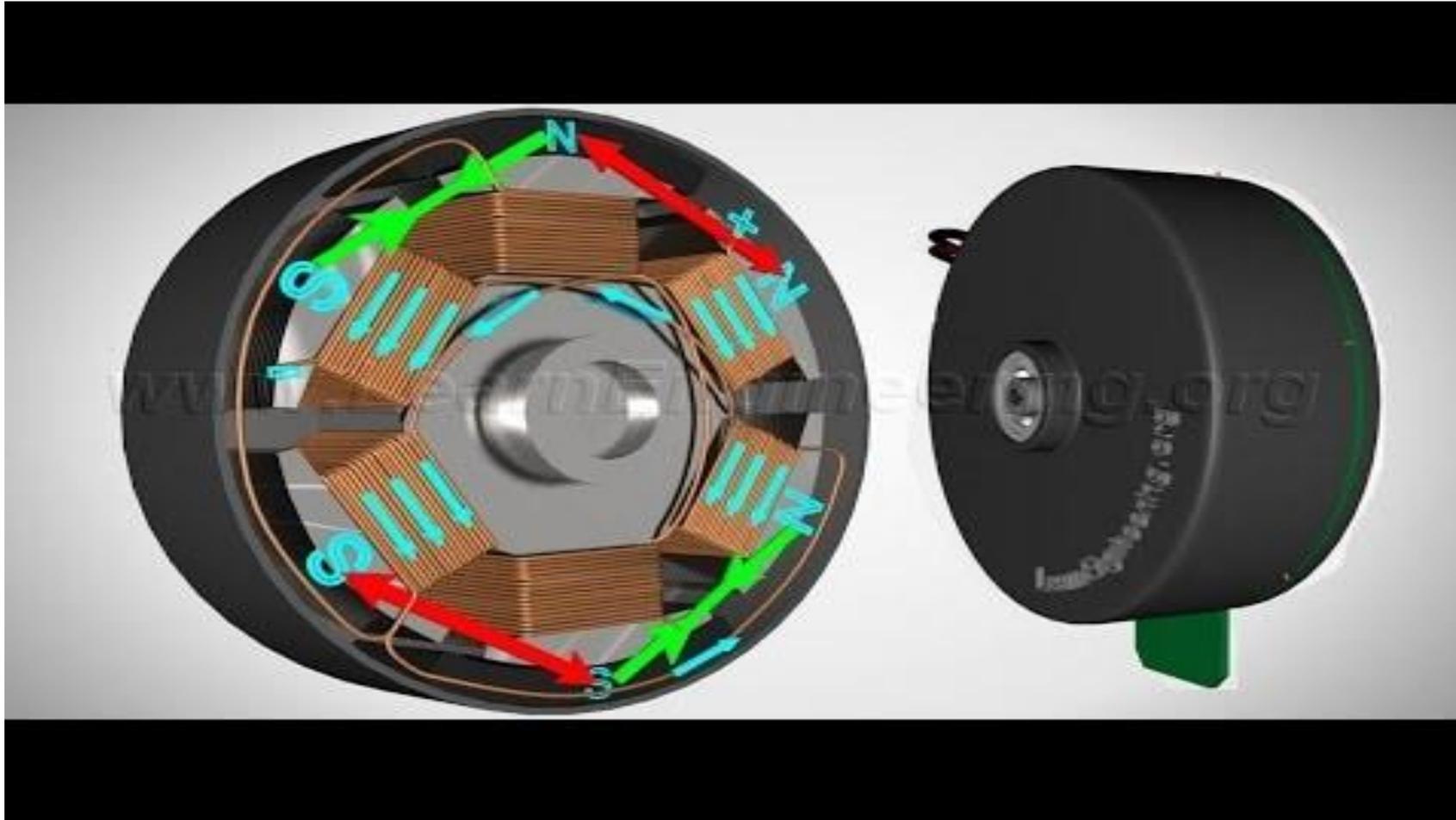
# Brushless DC Motors

- In the brushless motor, the rotor and the stator are swapped. Unlike the traditional DC motor, the stator of the brushless motor consists of windings, whereas the rotor comprises permanent magnets.
- In the brushless motor, the commutation of currents is performed with an electronic switching system, by measuring the rotor position using a position sensor.
- Depending on the rotor position, currents are delivered to the corresponding windings through electronic switching circuits.
- The principle of torque generation remains the same, and the torque-speed characteristics and other properties are mostly preserved. Therefore, the brushless motor is highly efficient with added reliability.





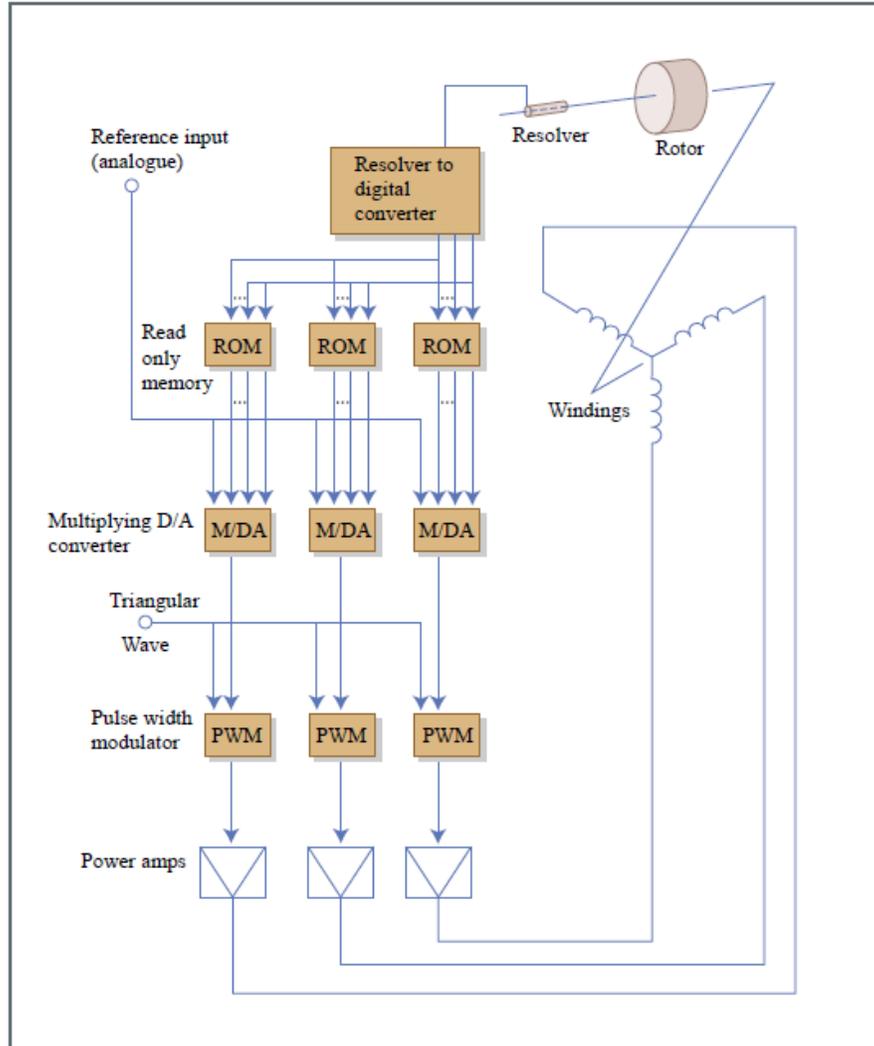
<https://www.youtube.com/watch?v=bCEiOnuODac>





# Brushless DC Motors

A common construction of the windings (armatures) is that of a three-phase windings, as shown in Figure



Let  $I_A$ ,  $I_B$  and  $I_C$  be individual currents flowing into the three windings (armatures) shown in the figure. These three currents are varies such that:

$$I_A = I_o \sin \theta$$

$$I_B = I_o \sin\left(\theta + \frac{2}{3} \pi\right)$$

$$I_C = I_o \sin\left(\theta + \frac{4}{3} \pi\right)$$

where  $I_o$  is the scalar magnitude of desired current, and  $\theta$  is the rotor position.



# Brushless DC Motors

The torque generated is the summation of the three torques generated at the three windings. Taking into account the angle between the magnetic field and the force generated at each air gap, we obtain

$$\tau_m = k_o \left[ I_A \sin \theta + I_B \sin \left( \theta + \frac{2}{3} \pi \right) + I_C \sin \left( \theta + \frac{4}{3} \pi \right) \right]$$

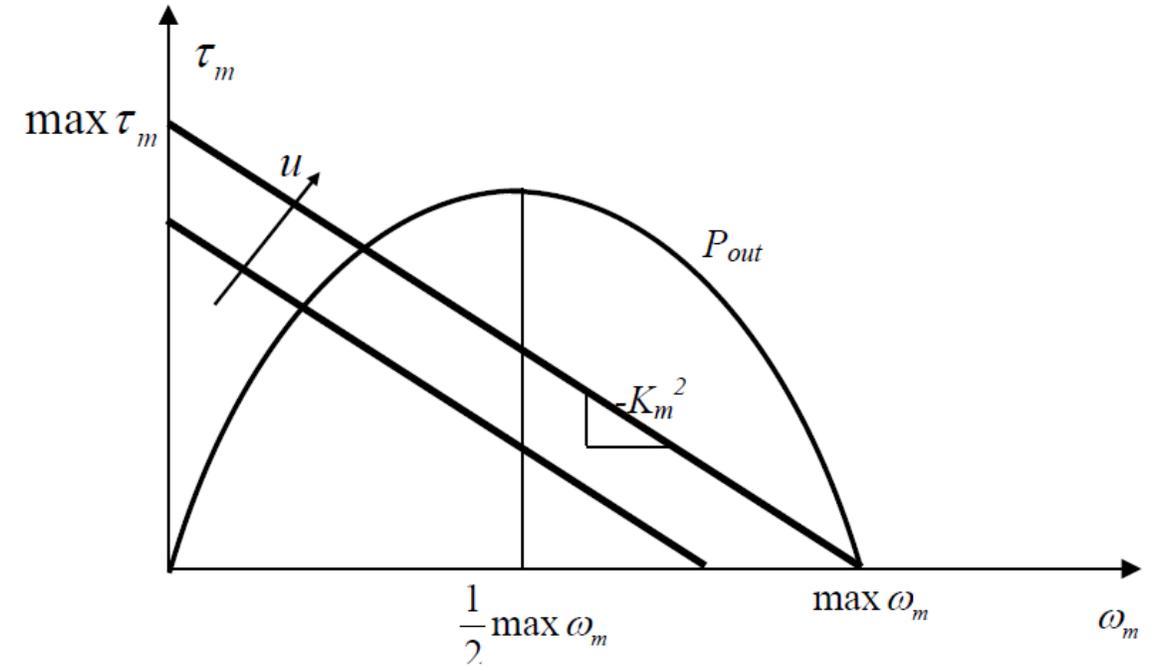
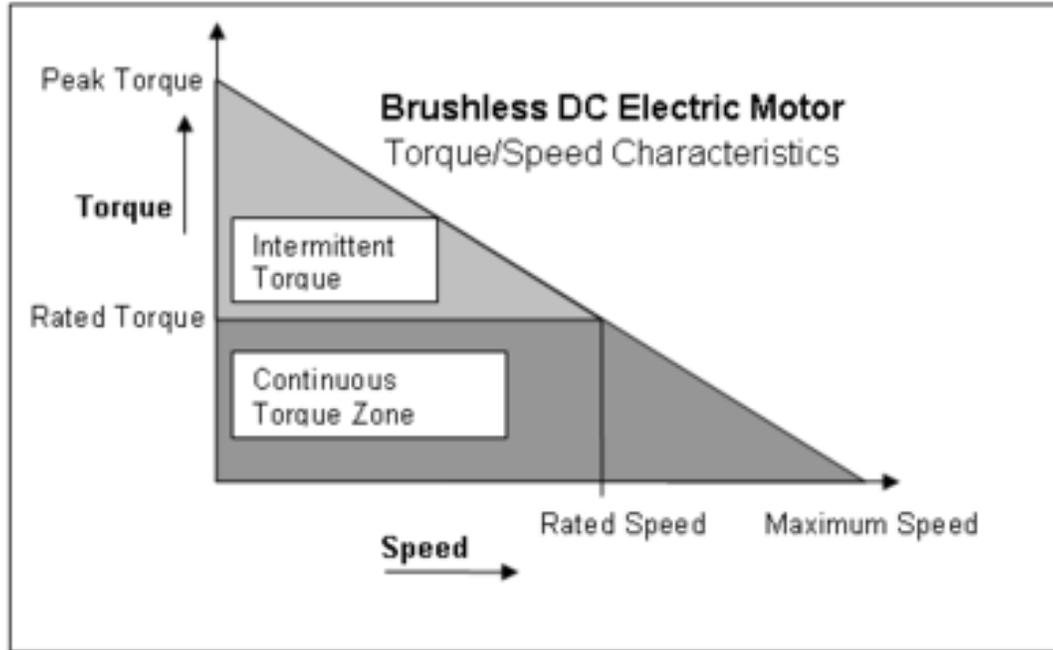
where  $k_o$  is a proportionality constant. Substituting eq. of the three currents it yields

$$\tau_m = \frac{2}{3} k_o I_o$$

The above expression indicates a linear relationship between the output torque and the scalar magnitude of the three currents. The torque-current characteristics of a brushless motor are apparently the same as the traditional DC motor.



# Brushless DC Motors





# The end!

Thank you for your Attention!!!

Any Questions?

